# FUN WITH FIBONACCI AT THE CHESS MATCH 

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As first noted by Hoggatt [1], the results of the world championship chess match held in Iceland this summer between Boris Spassky, U. S. S. R., and Bobby Fischer, U. S. A., were heavily influenced by Fibonacci and Lucas numbers (underlined in what follows). Fischer started as a $\underline{5}$ to $\underline{2}$ favorite with the British bookmakers, and Jimmy "The Greek" Snyder in Las Vegas, Nevada, gave Fischer 6 to $\underline{5}$ which was changed to $\underline{8}$ to $\underline{5}$ as reported in Time, August 21. After the first and second games, Fischer won all games whose numbers are Fibonacci numbers, while all games numbered by Lucas numbers (apart from 3) were either draws or wins for Spassky. They played 7 consecutive draws (games 14 to 20 inclusive); there were $\underline{8}$ consecutive games where Fischer was $\underline{3}$ games ahead of Spassky, and Fischer was 4 games ahead of Spassky when he won the match. The 21 games ended with 11 draws, $\underline{7}$ wins for Fischer and $\underline{3}$ for Spassky, with a prize split of $\underline{5 / 8}$ for Fischer and $\underline{3 / 8}$ for Spassky, or, a gold ratio of $\underline{5}$ to $\underline{3}$.

There were $\underline{8}$ occasions when both scores were positive integers. After $\underline{5}$ of the games ( $\underline{1}, \underline{2}, \underline{3}, \underline{8}, \underline{13}$ ), both scores were Fibonacci numbers and consecutive Fibonacci numbers in $\underline{4}$ of those cases; in $\underline{2}$ cases ( $\underline{3}$ and $\underline{7}$ ), both scores were Lucas numbers; in $\underline{8}$ cases $\underline{1}, \underline{2}, \underline{3}$, 7, $\underline{8}, 12,13,19$ - where there are 4 Lucas-numbered and 5 Fibonacci-numbered games listed), the scores were each Fibonacci or Lucas numbers. In $\underline{7}$ cases ( $\underline{1}, \underline{2}, \underline{3}, \underline{7}, \underline{8}, 12$, 19) there were one Lucas and one Fibonacci number.

Even when the scores were non-integral, all was well with Fibonacci. It is well-known that there are several ways to round off numbers ending with $1 / 2$. If one score is rounded up and the other rounded down, in $\underline{8}$ cases $(\underline{4}, \underline{5}, 6,9,10, \underline{11}, \underline{18}, \underline{21})$ the scores will both be either a Fibonacci or a Lucas number; in $\underline{3}$ cases (4, $\underline{5}, \underline{21}$ ) the scores are both distinct Fibonacci numbers, being consecutive Fibonacci numbers in games $\underline{5}$ and $\underline{21}$ and making Fibonacci numbers $\underline{2}$ ways for the scores from game $\underline{4}$; in $\underline{5}$ cases ( $\underline{4}, 6,10, \underline{11}, \underline{18}$ ) both scores will be Lucas numbers with positive subscripts, being consecutive Lucas numbers in games $\underline{4}, \underline{11}$, and $\underline{18}$; in $\underline{3}$ cases $\underline{4}, \underline{5}, 6)$ both scores are both Fibonacci and Lucas numbers for both ways to round up and down. Notice that, in $\underline{3}$ cases ( $\underline{4}, \underline{5}, \underline{21}$ ), the scores are both distinct Fibonacci numbers while in 4 other cases ( $6,10,11,18$ ) the scores are both Lucas numbers. There were $\underline{5}$ games $(\underline{4}, 6,10, \underline{11}, \underline{18})$ after which if both scores were rounded up, they would both be either (distinct) Fibonacci or Lucas numbers, and again $\underline{3}$ cases $(\underline{4}, \underline{5}, 6)$ where both scores are both Fibonacci and Lucas numbers. Similarly, there are $\underline{5}$ cases ( $\underline{4}$, $6,9,14,20$ ) in which if both scores were rounded down, both numbers resulting are either
(distinct) Fibonacci or Lucas numbers. Game 5 yields tied Fibonacci and/or Lucas numbers upon rounding both scores either up or down. Further, the non-integral entries, upon dropping the fraction $\underline{1 / 2}$, yield 13 entries which are Fibonacci numbers, and 11 which are Lucas numbers, made up of 7 distinct numbers; further, 8 are both Fibonacci and Lucas numbers, while $\underline{8}$ are separately Fibonacci or Lucas numbers. Games 4, 5, and 6 make two scores which are each both Fibonacci and Lucas numbers for all $\underline{3}$ ways to round off the scores, and a Fibonacci or Lucas number of game numbers are underlined in each case listed in this paragraph. There were 11 games where the scores were non-integral. Writing all non-integral scores as improper fractions yields 8 Fibonacci only numerators and 7 Lucas only numerators and 1 which is both.

The ratio of the two scores equalled the ratio of two positive Fibonacci numbers after $\underline{7}$ of the games ( $\underline{3}, \underline{4}, \underline{5}, \underline{7}, \underline{13}, 15,19$ ), while after $\underline{3}$ of the games ( $\underline{5}, \underline{7}, \underline{8}$ ) the ratio of the scores was equal to the ratio of two Lucas numbers with positive subscripts. After the $\underline{8}$ games $\underline{1}, \underline{2}, \underline{3}, \underline{5}, 6, \underline{8}, \underline{13}$ and $\underline{21}$, the ratios of the scores are the ratios of two Fibonacci numbers, and after the $\underline{7}$ games $\underline{3}, \underline{4}, 6, \underline{7}, 10, \underline{11}$, and $\underline{18}$ the scores are two Lucas numbers if Spassky's score is rounded down and Fischer's rounded up. Further, in each list of games cited, a Fibonacci or Lucas number of games is underlined.

Note that game $\underline{5}$ fits all $\underline{5}$ criteria given for non-integral scores. Lastly, there is exactly one game, game 16, whose scores fit into none of the preceding patterns, again a Fibonacci count. The scores of this remarkable match follow.

SCORES IN WORLD CHAMPIONSHIP CHESS MATCH
(Scoring: Win, 1 point; Draw, $1 / 2$ point)

| Winner | Game | Spassky | Fischer | Winner | Game | Spassky | Fischer |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 1 | 0 | D | 12 | 5 | 7 |
| S | 2 | 2 | 0 | F | 13 | 5 | 8 |
| F | 3 | 2 | 1 | D | 14 | $5-1 / 2$ | $8-1 / 2$ |
| D | 4 | $2-1 / 2$ | $1-1 / 2$ | D | 15 | 6 | 9 |
| F | 5 | $2-1 / 2$ | $2-1 / 2$ | D | 16 | $6-1 / 2$ | $9-1 / 2$ |
| F | 6 | $2-1 / 2$ | $3-1 / 2$ | D | 17 | 7 | 10 |
| D | 7 | 3 | 4 | D | 18 | $7-1 / 2$ | $10-1 / 2$ |
| F | 8 | 3 | 5 | D | 19 | 8 | 11 |
| D | 9 | $3-1 / 2$ | $5-1 / 2$ | D | 20 | $8-1 / 2$ | $11-1 / 2$ |
| F | 10 | $3-1 / 2$ | $6-1 / 2$ | F | 21 | $8-1 / 2$ | $12-1 / 2$ |
| S | 11 | $4-1 / 2$ | $6-1 / 2$ |  |  |  |  |

Making a different count, notice that, out of 42 scores occurring, if fractions are discarded, Lucas or Fibonacci numbers occur 34 times while there are only $\underline{8}$ occurrences of non-Fibonacci, non-Lucas numbers, and, further, each score occurs a Fibonacci or Lucas number of times. If both scores are rounded up, each score occurs a Lucas or Fibonacci number of times.
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