In Figure 2, we see that the "golden sequence," $1, \phi, \phi^2, \cdots, \phi^n, \cdots$ has its geometric analogue in the sequence of altitudes P_0B_0, P_1B_1, P_2B_2, $\cdots$, P_nB_n, $\cdots$. In that same figure, the sequence of altitudes P_1B_0, P_2B_1, P_3B_2, $\cdots$, $P_{n+1}B_n$, $\cdots$ suggests a second sequence which is also golden: $\phi^{1/2}, \phi^{3/2}, \phi^{5/2}, \cdots$. (That this sequence is geometric is clear; property (viii) demonstrates that it is also a Fibonacci sequence.) The following are additional extensions of the golden sequence suggested by the appropriate sequences of altitudes in Figs. 2 and 3 (we include the above two for completeness):

(1) $1, \phi, \phi^2, \phi^3, \cdots, \phi^{n-1}, \cdots$ (Golden Sequence)

(2) $1/\phi, 1/\phi^2, 1/\phi^3, \cdots, 1/\phi^n, \cdots$

(3) $\cdots, 1/\phi^n, 1/\phi^{n-1}, \cdots, 1/\phi, 1, \phi, \phi^2, \phi^3, \cdots, \phi^{n-1}, \cdots$

(4) $\phi^{1/2}, \phi^{3/2}, \phi^{5/2}, \cdots, \phi^{(2n-1)/2}, \cdots$

(5) $\phi^{-1/2}, \phi^{-3/2}, \phi^{-5/2}, \cdots, \phi^{-(2n-1)/2}, \cdots$

(6) $\cdots \phi^{-(2n-1)/2}, \cdots, \phi^{-1/2}, \phi^{1/2}, \phi^{3/2}, \phi^{5/2}, \cdots, \phi^{(2n-1)/2}, \cdots$

As a final remark, we consider the sequence suggested by the complete sequence of altitudes in Fig. 3:

$$\cdots, P'_nB'_{n+1}, P'_nB'_n, P'_{n-1}B'_n, P'_{n-1}B'_{n-1}, \cdots, P'_1B'_2, P'_1B'_1, P_0B'_1, P_0B_0,$$
$$P_1B_0, P_1B_1, \cdots, P_nB_{n-1}, P_nB_n, \cdots .$$

This geometric sequence, with ratio $\phi^{1/2}$, is evidently

$$\cdots, \phi^{-(2n+1)/2}, \phi^{-n}, \phi^{-(2n-1)/2}, \phi^{-(n-1)}, \cdots, \phi^{-3/2}, \phi^{-1}, \phi^{-1/2}, 1,$$
$$\phi^{1/2}, \phi, \cdots, \phi^{(2n-1)/2}, \phi^n, \cdots .$$

Although this is not a Fibonacci sequence (and Hence, not golden), it contains each of the golden sequences, (1)-(6), as subsequences, and has the easily verified property that any subsequence consisting of alternate terms of the sequence, is in fact, a golden sequence.

REFERENCE

1. H. E. Huntley, The Divine Proportion, Dover, New York, 1970.

ERRATA

In "Ye Olde Fibonacci Curiosity Shoppe," appearing in Vol. 10, No. 4, October, 1972, please make the following changes:

Page 443: In the first line of the second paragraph, insert the word "ten" between "of" and "consecutive," so that it reads "... the sum of the squares of ten consecutive Fibonacci numbers is always divisible by $F_{10} = 55$."