Let $S(X^2)_q$ symbolize the sum of the digits of $X^2$ on the base $q$. For example, $S(9^2)_5 = S(14^2)_5 = 5$ since $9^2 = 31$. The following is a method for finding $q$ such that $S(X^2)_q = X$ when $X$ is given. For example $S(7^2)_8 = 7$ since $7^2 = 49$.

Step 1. List all the factors of $X$ except $X$ itself.
Step 2. List all the factors of $X - 1$.
Step 3. Multiply each factor of $X$ by one of the factors of $X - 1$, discarding all products greater than $X - 1$. The retained products are the ten's digits of the $X^2_q$ that we seek.
Step 4. The unit's digits can be obtained by simple subtraction of the quantities in three from $X$. 
Step 5. $q$ can now be computed by simple arithmetic.

Example. $S(21^2)_q = 21$. Find all values of $q$.

Step I: 

$$
\begin{array}{ccc}
1 & 3 & 7 \\
2 & 2 & 4 & 5 & 10 & 20 \\
3 & 2 & 4 & 5 & 10 & 20 \\
7 & 6 & 12 & 15 \\
\end{array}
$$

Step II: 

$$
\begin{array}{ccc}
1 & 20 & 2(19) & 4(17) & 5(16) & 10(11) & 20(1) \\
3 & 18 & 6(15) & 12(9) & 15(6) & 7(14) & 14(7) \\
\end{array}
$$

The quantities in parentheses are the unit's digits.

Step V: For example, for $5(16), 5b + 16 = 441$ in base ten so that $b = 85$ expressed as a base ten number. The bases taken in order are

$$
\begin{array}{cccccccc}
421 & 211 & 106 & 85 & 43 & 22 \\
141 & 71 & 36 & 29 & 61 & 31 \\
\end{array}
$$

The problem is: Why does this method work?

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If eleven alternate terms of any Fibonacci sequence are added and divided by $L_{14}(199)$, the result is the middle term of the group of eleven terms added together.

Example. Using the series beginning $1, 4, \cdot \cdot \cdot$,

$$
157 + 411 + 1076 + 2817 + 7375 + 19308 + 50549 + 132339 + 346468 + 907065 + 2374727 = 3942292
$$

Dividing by 199 gives 19308.

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