
FIBONACCI SUMMATIONS INVOLVING A POWER OF A RATIONAL NUMBER

SUMMARY

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The formulas pertain to generalized Fibonacci numbers with given $T_1$ and $T_2$ and with

$$T_{n+1} = T_n + T_{n-1}$$

and with generalized Lucas numbers defined by

$$V_n = T_{n+1} + T_{n-1}$$

Starting with a finite difference relation such as

$$\Delta (b/a)^k T_{2k+2} = (b/a)^{k+1} T_{2k+2} (bT_{2k+4} - aT_{2k})$$

values of $b$ and $a$ are selected which lead to a single generalized Fibonacci or Lucas number for the term in parentheses. Thus for $b = 2$, $a = 13$, the quantity in parentheses is $3T_{2k-3}$. Using the finite difference approach leads to a formula

$$\sum_{k=1}^{n} (2/13)^k T_{2k} T_{2k+5} = (1/3) \left[ a^{n+1} T_{2n+5} T_{2n+7} - 2T_5 T_7 \right].$$

Formulas are also developed with terms in the denominator.

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