POWER SERIES AND CYCLIC DECIMALS

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There is an interesting relation between series based on the powers of an integer, and infinitely repeating decimal reciprocals whereby the sum of the powers of a single integer give not one, but two reciprocals. Figures 1 and 2 illustrate this in the case of the two integers 3 and 19, which yield respectively the decimal reciprocals *1/29, 1/7;* and *1/189, 1/81.* The left-hand member in each instance starts at the decimal point and develops (in reverse) to the left. Although it is obviously not a decimal, it is purely cyclic, and has the repetend of its decimal version. Since shifting the decimal by a suitable divisor rectifies this, and for the sake of simplicity, it is treated here as a decimal.

If M is any integer having k digits, the following equations apply:

$$(1) \qquad 1/(10M-1) = \sum_{n=1}^{\infty} M^{n-1} \times 10^{n-1}$$

and

$$(2) \qquad 1/(10^k - M) = \sum_{n=1}^{\infty} M^{n-1} \times 10^{-kn} \ .$$

Equation (1) is limited by the expression $(10M-1)$ to a fraction having a denominator with the last digit 9, and will thus be odd and yield a cyclic decimal fraction having a repetend with the terminal digit 1. Equation (2) is limited by the expression $(10^k - M)$ to a denominator which is the complement of M and will thus be odd, or even, and will not be limited as to type of repeating decimal. In the preparation of Figs. 1 and 2, zeros not contributing to the relations shown have been omitted.

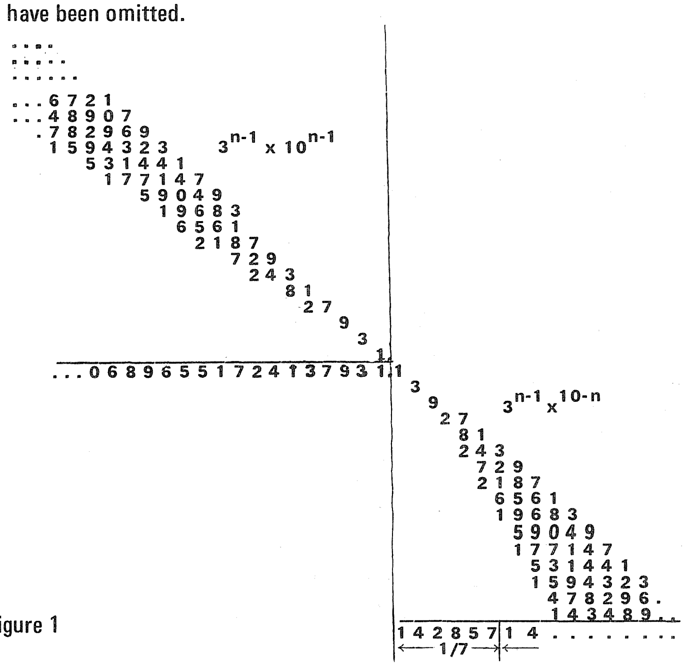

Figure 1

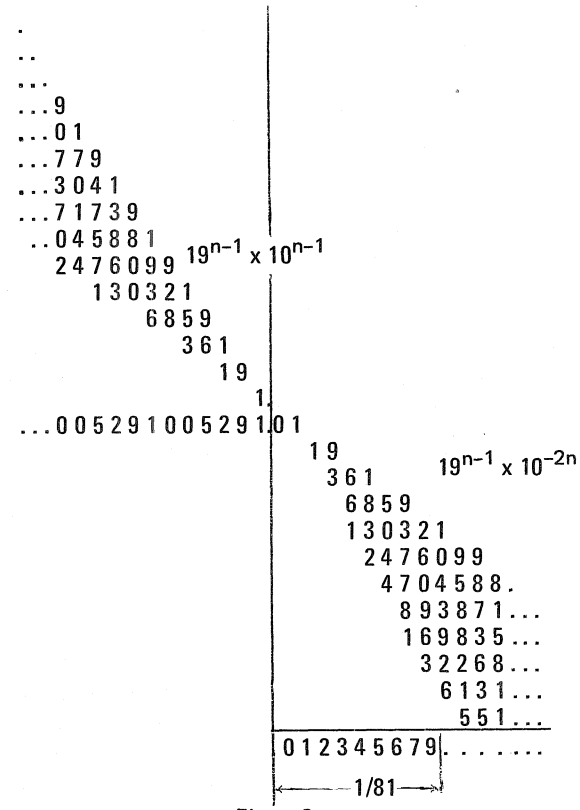

Figure 2

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ON GENERATING FUNCTIONS FOR POWERS OF A GENERALIZED SEQUENCE OF NUMBERS

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GENERATING FUNCTIONS

For the record, some results are presented here which arose many years ago (1965) in connection with the author's paper [3]. Familiarity with the notation and results of Carlitz [1], Riordan [6], and the author [2], [3] and [4], are assumed in the interests of brevity. Note, however, that h_n in [3] has been replaced by H_n to avoid ambiguity. Our results and techniques parallel those of Riordan.

Calculations yield

$$(1)\qquad \left\{\begin{aligned} H_n^2 - 3H_{n-1}^2 + H_{n-2}^2 &= 2(-1)^n e \\ H_n^3 - 4H_{n-1}^3 - H_{n-2}^3 &= 3(-1)^n eH_{n-1} \\ H_n^4 - 7H_{n-1}^4 + H_{n-2}^4 &= 2e^2 + 8(-1)^n eH_{n-1}^2 \\ H_n^5 - 11H_{n-1}^5 - H_{n-2}^5 &= 5e^2 H_{n-1} + 15(-1)^n eH_{n-1}^3 \,. \end{aligned}\right. \qquad (e = r^2 - rs - s^2)$$

and so on. Corresponding generating functions for the k^{th} power of H_n,

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