THE GENERAL LAW OF QUADRATIC RECIPROCITY

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Extend the definition of the Jacobi Symbol to include values for negative second entry as follows:
If a is an integer and p is an odd prime, set

$$(a/p) \equiv a^{(p-1)/2} \pmod{p}$$

and

$$(a/p) = 0 \text{ or } \pm 1 .$$

Set

$$(a/1) = 1.$$

If b is an odd integer, set

$$(a/b_1 b_2) = (a/b_1)(a/b_2).$$

Set

$$(0/-1) = 0.$$

Set

$$(-1/-1) = -1.$$

There is another way of defining negative second entry in the Jacobi Symbol, which is based upon

$$(-1/-1) = 1.$$

This method is given in [1, p. 38, Exercise IX, 5].

The Jacobi Symbol is only a definition and not a theorem; therefore it can be arbitrary as long as it satisfies two requirements: First, it must be consistent and, secondly, it must represent mathematical results clearly and elegantly. The definition given in this paper is superior from the second point of view. For example, with

$$(-1/-1) = 1$$

it is difficult to express the periodicity of the second entry. In fact, much of that periodicity is lost. But, with

$$(-1/-1) = -1,$$

the result is clearly stated in Corollary 2.

All of the known and proven properties of the Jacobi Symbol are retained in the extended definition (see [1, pp. 36–39] and [2, pp. 77–80]).

This refers in particular to the multiplicavity of the first entry, which is easily proved for negative second entry. Then

$$(a_1 a_2/b) = (a_1/b)(a_2/b)$$

and

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