Extend the definition of the Jacobi Symbol to include values for negative second entry as follows:
If \(a\) is an integer and \(p\) is an odd prime, set
\[
(a/p) = a^{(p-1)/2} \pmod{p}
\]
and
\[
(a/p) = 0 \text{ or } \pm 1.
\]
Set
\[
(a/1) = 1.
\]
If \(b\) is an odd integer, set
\[
(a/b,b_1) = (a/b_1)(a/b_2).
\]
Set
\[
(0/-1) = 0.
\]
Set
\[
(-1/-1) = -1.
\]
There is another way of defining negative second entry in the Jacobi Symbol, which is based upon
\[
(-1/-1) = 1.
\]
This method is given in [1, p. 38, Exercise IX, 5].
The Jacobi Symbol is only a definition and not a theorem; therefore it can be arbitrary as long as it satisfies two requirements: First, it must be consistent and, secondly, it must represent mathematical results clearly and elegantly. The definition given in this paper is superior from the second point of view. For example, with
\[
(-1/-1) = 1
\]
it is difficult to express the periodicity of the second entry. In fact, much of that periodicity is lost. But, with
\[
(-1/-1) = -1,
\]
the result is clearly stated in Corollary 2.
All of the known and proven properties of the Jacobi Symbol are retained in the extended definition (see [1, pp. 36–39] and [2, pp. 77–80]).
This refers in particular to the multiplicativity of the first entry, which is easily proved for negative second entry.
Then
\[
(a_1a_2/b) = (a_1/b)(a_2/b)
\]
and

[Continued on page 321.]