It then follows that the asymptotic density of C, and hence B, is 1. We have thus proved the following theorem.

Theorem 2. The probability of a random choice of a base $g \geq 3$ not yielding a solution to the Generalized Problem is 1.

In light of this theorem it seems that the choice of the base 10 in the problem as originally stated was a wise choice! We leave as an entertaining problem for the reader the question of the identity of the bases g less than 100 for which there is a solution.

We have shown that in some sense A has far fewer elements than B. But is A finite or infinite? If $g \equiv 3 \pmod 4$ is a prime and $p = g^2 - g - 1$ is also a prime, then $p \equiv 1 \pmod 4$ and

$$\left(\frac{g}{p}\right) = \left(\frac{p}{g}\right) = \left(\frac{-1}{g}\right) = -1.$$

Hence $g^t \equiv -1 \pmod p$ has a solution and $g \in A$. We note that Schinzel's Conjecture H [2] implies there are infinitely many primes $g \equiv 3 \pmod 4$ for which $g^2 - g - 1$ is also prime. Hence if this famous conjecture is true it follows that our set A is infinite.

REFERENCES

1. J. A. Hunter, Problem 301, *J. Recreational Math.*, 6 (4), Fall 1973, p. 308.
2. A. Schinzel and W. Sierpiński, "Sur certaines hypothèses concernant les nombres premiers," *Acta Arith.* 4 (1958), pp. 185–208.

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[Continued from P. 330.]

$$\left(\frac{(-1/a)}{(-1/b)}\right) = (-1)^{(a-1)(b-1)/4} = 1$$

if and only if $a \equiv 1 \pmod 4$ and/or $b \equiv 1 \pmod 4$.

If $A = \pm 1$ and $B = \pm 1$ are logical variables, then the sixteen functions of those variables are given by $\pm 1, \pm A, \pm B, \pm AB$ and $\pm(\pm A/\pm B)$. This is a result that cannot be obtained with the definition $(-1/-1) = 1$. If $A = (-1/b)$ and $B = (-2/b)$, then the logical functions of A and B give the congruence of b modulo 8. For example,

$$(A/B) = (-1)^{(b^3-b^2+7b-7)/16} = 1$$

if and only if $b \equiv 1, 3$ or $5 \pmod 8$. The function -1 is a null function which cannot occur.

If $b = \pm p_1 p_2 \cdots p_k$ with p_i not necessarily distinct, and n is the number of p_i for which $(a/p) = -1$, then

$$(ab) = \left(\frac{(a/-1)}{(b/-1)}\right)(-1)^n .$$

Theorem. If $ab \equiv 1 \pmod 2$ and $(a,b) = 1$, then

$$(a/b)(b/a) = \left(\frac{(a/-1)}{(b/-1)}\right)\left(\frac{(-1/a)}{(-1/b)}\right) .$$

In other words,

$$(a/b)(b/a) = 1$$

if and only if ((a is positive and/or b is positive) and ($a \equiv 1 \pmod 4$ and/or $b \equiv 1 \pmod 4$)) or (a is negative and b is negative and $a \equiv -1 \pmod 4$ and $b \equiv -1 \pmod 4$).

Proof.

$$((-1/a)/(-1/b)) = -1$$

if and only if

$$(-1/a) = (-1/b) = -1;$$

$$((-1/-a)/(-1/b)) = -1$$

[Continued on P. 336.]