$$Q = \begin{pmatrix} b & 1 \\ 1 & 0 \end{pmatrix}, \qquad Q^n = \begin{pmatrix} U_{n+1} & U_n \\ U_n & U_{n-1} \end{pmatrix} .$$

Since det $Q^n = (\det Q)^n = (-1)^n$, we have

(34) $$U_{n+1}U_{n-1} - U_n^2 = (-1)^n .$$

Using $Q^{m+n} = Q^m Q^n$ and equating elements in the upper left gives us

(35) $$U_{m+n+1} = U_{m+1}U_{n+1} + U_m U_n$$

(36) $$U_{2n+1} = U_{n+1}^2 + U_n^2 .$$

Many other identities can be found in the same way. Note that the characteristic polynomial of Q is $x^2 - bx - 1 = 0$.

Summation identities can also be generalized [1], [2], as, for example,

(37) $$U_0 + U_1 + U_2 + \cdots + U_n = (U_n + U_{n+1} - 1)/b$$

(38) $$V_0 + V_1 + V_2 + \cdots + V_n = (V_n + V_{n+1} + b - 2)/b$$

(39) $$U_0^2 + U_1^2 + U_2^2 + \cdots + U_n^2 = (U_n U_{n+1})/b .$$

The reader is left to see what other identities he can find which hold for the general sequence.

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2. A. F. Horadam, "Pell Identities," *The Fibonacci Quarterly,* Vol. 9, No. 3 (April 1971), pp. 245–252, 263.
3. Marjorie Bicknell, "A Primer for the Fibonacci Numbers: Part VII, An Introduction to Fibonacci Polynomials and Their Divisibility Properties," *The Fibonacci Quarterly,* Vol. 8, No. 4 (Oct. 1970), pp. 407–420.
4. Joseph A. Raab, "A Generalization of the Connection Between the Fibonacci Sequence and Pascal's Triangle," *The Fibonacci Quarterly,* Vol. 1, No. 3 (Oct. 1963), pp. 21–31.

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[Continued from P. 344.]

Corollary 2. If $ab \equiv 1 \pmod 2$ and $(a,b) = 1$, and if $b_1 \equiv b_2 \pmod{2a}$, then

$$(a/b_1b_2) = \left(\frac{(-1/a)}{(-1/b_1b_2)} \right) .$$

In other words,

$$(a/b_1b_2) = 1$$

if and only if $a \equiv 1 \pmod 4$ and/or $b_1b_2 \equiv 1 \pmod 4$.

Proof. From (b_1b_2/a), $(-b_1b_2/a)$, $(b_1b_2/-a)$ and $(-b_1b_2/-a)$, the following results can be obtained by quadratic reciprocity:

[Continued on P. 384.]