6. Coefficient of $-2m$

$$F_s^3F_{s-1}+F_{s-1}^2F_sF_{s+1} = F_{s-1}F_s[F_s^2+F_{s-1}F_{s+1}] = F_{s-1}F_s[F_s(F_{s+2}-F_{s+1})+F_{s-1}F_{s+1}]$$

$$= F_{s-1}F_s[F_sF_{s+2}-F_{s+1}(F_s-F_{s-1})] = F_{s-1}F_s(F_sF_{s+2}-F_{s+1}F_{s-2}).$$

$$(F_1^2+F_2^2+\cdots+F_{s-1}^2)(1+2F_1F_2+2F_sF_3+\cdots+2F_{s-1}F_s = F_{s-1}F_s[F_sF_{s+2}-F_{s+1}F_{s-2}]$$

In proving this identity the following Fibonacci identities were used:

(a) $$1+2F_1F_2+\cdots+2F_{s-1}F_s = F_sF_{s+2}-F_{s+1}F_{s-2}$$

(b) $$F_1^2+F_2^2+\cdots+F_s^2 = F_{s-1}F_s$$

(c) $$F_{s-1}F_{s+1} = F_s^2+(-1)^s\ .$$

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A MORE GENERAL FIBONACCI MULTIGRADE

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In a recent article I gave examples of multigrades based on Fibonacci series in which

$$F_{n+2} = F_{n+1}+F_n\ .$$

Here I first give a more general multigrade for series in which

$$F_{n+2} = yF_{n+1}+xF_n\,.$$

Consider

1 3 7 17 47 (where $x = 1, y = 2$).

By inspection we notice that

$$1^m+3^m+3^m+7^m = 0^m+4^m+4^m+6^m$$

$$3^m+7^m+7^m+17^m = 0^m+10^m+10^m+14^m\ \text{, etc.}$$

(where $m = 1, 2$).

We can look at other series of a like kind:

1 3 10 33 109 (where $x = 1,\ y = 3$).

Here

$$1^m+3^m+3^m+3^m+10^m+10^m = 0^m+0^m+7^m+7^m+7^m+9^m$$

$$3^m+10^m+10^m+10^m+33^m+33^m = 0^m+0^m+23^m+23^m+23^m+30^m,\ \text{etc.}$$

(where $m = 1, 2$)

1 3 11 39 139 (where $x = 2,\ y = 3$).

Here

$$1^m+1^m+3^m+3^m+3^m+11^m+11^m+11^m = 0^m+0^m+0^m+8^m+8^m+8^m+10^m+10^m$$

$$3^m+3^m+11^m+11^m+11^m+39^m+39^m+39^m = 0^m+0^m+0^m+28^m+28^m+28^m+36^m+36^m,\ \text{etc.}$$

(where $m = 1, 2$)

The general series

$$a \qquad b \qquad ax+by \qquad bx+axy+by^2$$

gives

$$x(a)^m+y(b)^m+(x+y-2)(ax+by)^m = (x+y-2)0^m+y(ax+by-b)^m+x(ax+by-a)^m$$

(where $m = 1, 2$).

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