ON CONTINUED FRACTION EXPANSIONS WHOSE ELEMENTS ARE ALL ONES

FEB. 1976

6. Coefficient of \(-2m\)
\[
F_s^3 F_{s-1} + F_{s-1}^2 F_s F_{s+1} = F_{s-1} F_s [F_{s+2}^2 + F_{s+1} F_s + F_{s+1}] = F_{s-1} F_s [F_s (F_{s+2} - F_{s+1}) + F_{s+1} F_{s+1}]
\]
\[= F_{s-1} F_s [F_s F_{s+2} - F_{s+1} (F_s - F_{s-1})] = F_{s-1} F_s (F_s F_{s+2} - F_{s+1} F_{s-2}).
\]
\[(F_1^2 + F_2^2 + \ldots + F_s^2) (1 + 2F_1 F_2 + 2F_2 F_3 + \ldots + 2F_{s-1} F_s) = F_{s-1} F_s [F_s F_{s+2} - F_{s+1} F_{s-2}]
\]

In proving this identity the following Fibonacci identities were used:
(a) \(1 + 2F_1 F_2 + \ldots + 2F_{s-1} F_s = F_s F_{s+2} - F_{s+1} F_{s-2}\)
(b) \(F_1^2 + F_2^2 + \ldots + F_s^2 = F_{s-1} F_s\)
(c) \(F_{s-1} F_{s+1} = F_s^2 + (-1)^s\).

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A MORE GENERAL FIBONACCI MULTIGRADE

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In a recent article I gave examples of multigrades based on Fibonacci series in which
\(F_{n+2} = F_{n+1} + F_n\).

Here I first give a more general multigrade for series in which
\(F_{n+2} = y F_{n+1} + x F_n\).

Consider
\[
1 \ 3 \ 7 \ 17 \ 47 \quad (\text{where } x = 1, y = 2).
\]

By inspection we notice that
\[
1^m + 3^m + 7^m = 0^m + 4^m + 6^m
\]
\[3^m + 7^m + 17^m = 0^m + 10^m + 14^m, \quad \text{etc.}
\]
\[
(\text{where } m = 1, 2).
\]

We can look at other series of a like kind:
\[
1 \ 3 \ 10 \ 33 \ 109 \quad (\text{where } x = 1, y = 3).
\]

Here
\[
1^m + 3^m + 7^m + 10^m + 10^m = 0^m + 0^m + 7^m + 7^m + 17^m + 4^m + 6^m
\]
\[3^m + 10^m + 10^m + 10^m + 33^m + 33^m = 0^m + 0^m + 23^m + 23^m + 23^m + 23^m + 30^m, \quad \text{etc.}
\]
\[
(\text{where } m = 1, 2).
\]

Here
\[
1^m + 1^m + 3^m + 3^m + 3^m + 11^m + 11^m + 11^m = 0^m + 0^m + 0^m + 0^m + 0^m + 8^m + 8^m + 8^m + 10^m + 10^m
\]
\[3^m + 3^m + 11^m + 11^m + 11^m + 39^m + 39^m + 39^m = 0^m + 0^m + 0^m + 28^m + 28^m + 28^m + 28^m + 36^m + 36^m, \quad \text{etc.}
\]
\[
(\text{where } m = 1, 2).
\]

The general series
\begin{align*}
a & \ b & \ ax + by & \ bx + ay + by^2
\end{align*}

gives
\[
x(ax)^m + y(by)^m + (x + y - 2)(ax + by)^m = (x + y - 2)(ax + by)^m + x(ax + by - a)^m
\]
\[
(\text{where } m = 1, 2).
\]

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