REFERENCES

1. F. Chorlton, *Ordinary Differential and Difference Equations, Theory and Applications,* Princeton, N. J., Van Nostrand, 1965, Chapters 8–10.
2. H. Levy and F. Lessman, *Finite Difference Equations,* New York, MacMillan, 1961, Chapters 4–6.
3. C. H. Richardson, *An Introduction to the Calculus of Finite Differences,* Princeton, N. J., Van Nostrand, 1954, Chapter VI.
4. Chorlton, p. 186.
5. F. S. Macaulay, *The Algebraic Theory of Modular Systems,* London, Cambridge University Press, 1916, pp. 4-16.
6. R. G. Babb, "On the Order of Systems of Two Simultaneous Linear Difference Equations in Two Variables," M. Math Thesis, Univ. of Waterloo, Waterloo, Ontario, May 1974 (available by Xerox copy from the Fibonacci Association).

Continued from page 66. *******

If we add any quantity B to each term, the above becomes

$$(x^2+B)^m - 3[(x+1)^2+B]^m + 3[(x+2)^2+B]^m - [3[(x+2)^2-(x+1)^2]+x^2+B]^m$$

$$= -B^m - 3[x^2-4(x+1)^2+3(x+2)^2+B]^m + 3[x^2-3(x+1)^2-4(x+2)^2+B]^m + [3[(x+2)^2-(x+1)^2]+B]^m$$

(where $m = 1, 2$).

Finally, take the series in which

$$F_n = A_{n-1}F_{n-1} + A_{n-2}F_{n-2} \cdots A_2F_2 + A_1F_1.$$

We conjecture that

$$A_1F_1^m + A_2F_2^m + A_3F_3^m \cdots A_{n-2}F_{n-2}^m + A_{n-1}F_{n-1}^m + \left(\sum_1^{n-1} A - 2\right)F_n^m$$

(2)

$$= A_1(F_n-F_1)^m + A_2(F_n-F_2)^m + A_3(F_n-F_3)^m \cdots A_{n-2}(F_n-F_{n-2})^m + A_{n-1}(F_n-F_{n-1})^m + \left(\sum_1^{n-1} A-2\right)0^m$$

(where $m = 1, 2$).

Proof: When $m = 1$,

$$\text{L.H.S.} = \left(\sum_1^{n-1} A - 1\right)F_n.$$

When $m = 1$,

$$\text{R.H.S.} = (A_1+A_2+A_3\cdots A_{n-2}+A_{n-1})F_n - (A_1F_1+A_2F_2+A_3F_3\cdots A_{n-2}F_{n-2}+A_{n-1}F_{n-1}) = \left(\sum_1^{n-1} A-1\right)F_n.$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

When $m = 2$,

$$\text{L.H.S.} = A_1F_1^2 + A_2F_2^2 + A_3F_3^2 \cdots A_{n-2}F_{n-2}^2 + A_{n-1}F_{n-1}^2 + \left(\sum_1^{n-1} A-2\right)F_n^2.$$

When $m = 2$,

$$\begin{aligned}\text{R.H.S.} &= A_1F_n^2 - 2A_1F_1F_n + A_1F_1^2 + A_2F_n^2 - 2A_2F_2F_n + A_2F_2^2 \\ &\quad + A_3F_n^2 - 2A_3F_3F_n + A_3F_3^2 + \cdots \\ &\quad + A_{n-1}F_n^2 - 2A_{n-1}F_{n-1}F_n + A_{n-1}F_{n-1}^2 \\ &= \sum_1^{n-1} AF_n^2 - 2F_n\cdot F_n + A_1F_1^2 + A_2F_2^2 + A_3F_3^2\cdots A_{n-1}F_{n-1}^2 \\ &= \left[\sum_1^{n-1} A-2\right]F_n^2 + A_1F_1^2 + A_2F_2^2 + A_3F_3^2 \cdots A_{n-1}F_{n-1}^2 = \text{L.H.S.}\end{aligned}$$

If we add any quantity B to each term, we get

$$A_1(F_1+B)^m + A_2(F_2+B)^m + A_3(F_3+B)^m \cdots A_{n-2}(F_{n-2}+B)^m + A_{n-1}(F_{n-1}+B)^m + \left(\sum_1^{n-1} A-2\right)(F_n+B)^m$$

$$= A_1(F_n-F_1+B)^m + A_2(F_n-F_2+B)^m + A_3(F_n-F_3+B)^m \cdots A_{n-2}(F_n-F_{n-2}+B)^m + A_{n-1}(F_n-F_{n-1}+B)^m$$

$$+ \left(\sum_1^{n-1} A-2\right)B^m$$ (where $m = 1, 2$). *Continued on page 92.*