FIBONACCI RATIO IN ELECTRIC WAVE FILTERS

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In the classical theory of electric wave filters, a complete filter is composed of a series of sections in cascade or tandem, terminated at each end by a terminal half-section. Sections can be of T or π circuit configuration (sometimes called "mid-shunt" and "mid-series" sections).

Aside from the frequency selective properties of the filter, an important design requirement is that the image impedances at the input and output terminals shall be as nearly constant as possible throughout the greater part of the pass-band. For this reason the terminating half-sections are usually of the "m-derived" type, where m is a design parameter that can be chosen as anything in the interval between zero and unity, but is usually about 0.6 to satisfy the requirement of constant image impedance in the pass-band. This is the primary function of the terminating half-sections. Figure 1 shows a family of curves giving the variations in image impedance throughout the pass-band for various values of m. Notice that the curve for $m = 0.6$ results in a variation of the ordinate within $\pm 5\%$ over 75% of the pass-band, and provides a good approximation to constant image impedance over the useful range of frequency.

To demonstrate that the natural value for m is the inverse Fibonacci Ratio, $1/\phi$, where

$$\frac{1}{\phi} = \frac{\sqrt{5}-1}{2} = 0.618;$$

we use a low-pass T section as an example. Figure 2 shows a circuit diagram of such a mid-shunt section, together with a sketch of its frequency selective characteristic.

The salient features of the response function are the two frequencies f_c = cutoff and f_∞ = the frequency for infinite attenuation.

Letting the ratio $(f_c/f_\infty) = r$; the relation between m and r is the equation of a circle:

$$m^2 + r^2 = 1,$$

where m and r are both restricted to non-negative real values.

The design formulas are:

$$L_1 = mL_K \qquad\qquad L_K = \frac{R}{\pi f_c}$$

$$L_2 = \frac{1-m^2}{m}\;\frac{L_K}{4}$$

$$C_2 = mC_K \qquad\qquad C_K = \frac{1}{\pi R f_c},$$

where R is the load resistance at the terminals and therefore the desired image impedance of the terminating half-section. The circuit diagram of the m-derived half-section is shown in Fig. 3.

We notice that the coefficient involving m of the midshunt inductance L_2 is: $(1-m^2)/m$ and letting this coefficient equal unity gives:

$$1 - m^2 = m, \qquad m^2 + m - 1 = 0,$$

from which the positive real root is:

$$m = \frac{-1+\sqrt{1+4}}{2} = \frac{\sqrt{5}-1}{2} = 0.618 = \frac{1}{\phi}.$$

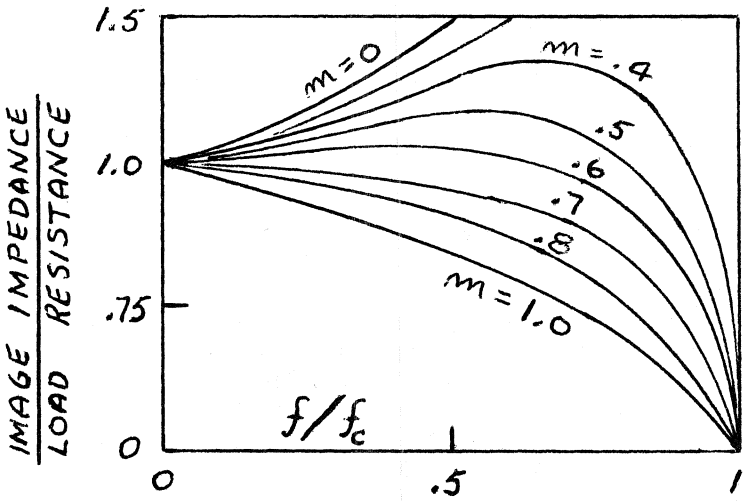

FIG. 1
IMAGE IMPEDANCE VS. FREQUENCY

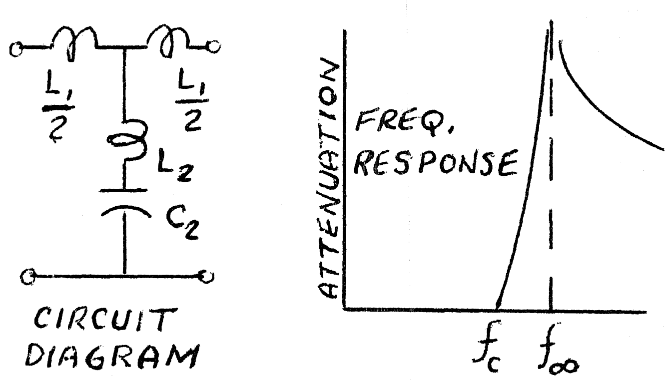

FIG 2: LOW-PASS T-SECTION

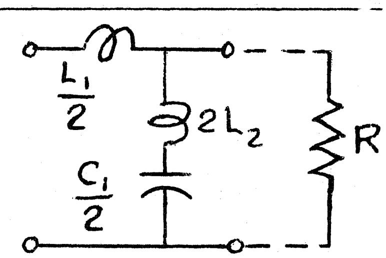

FIG. 3
CIRCUIT DIAGRAM OF m-DERIVED HALF-SECTION

Then:

$$r^2 = 1 - m^2 = 1 - \frac{1}{\phi^2} = 0.618 = \frac{1}{\phi} \quad , \quad \frac{1}{r} = \sqrt{\phi} = 1.272$$

This is substantially in agreement with a design rule that the frequency at infinite attenuation should be about 25% higher than the cutoff frequency.

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