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LETTER TO THE EDITOR

April 21, 1975

Dear Editor:

Following are some remarks on some formulas of Trumper [5].

Trumper has proved seven formulas of which the following is entirely characteristic

(1) $$F_nF_m - F_xF_{n+m-x} = (-1)^{m+1}F_{x-m}F_{n-x}\,.$$

He actually gives 13 formulas, but the duplicity arises from the trivial replacement of x by $-x$ in all but the seventh formula.

It is of interest to note that the formulas are not really new in the sense that they can all be gotten from the single formula

(2) $$F_{n+a}F_{n+b} - F_nF_{n+a+b} = (-1)^nF_nF_b$$

by use of the negative transformation

(3) $$F_{-n} = (-1)^{n+1}F_n\,.$$

For example, in (1) replace n by $n+x$ and m by $m+x$, and we have

$$F_{x+n}F_{x+m} - F_xF_{x+n+m} = (-1)^{m+x+1}F_{-m}F_n = (-1)^xF_mF_n\,,$$

the last step following by (3). But the formula is then simply a restatement of (2) with n replaced by x, a by n, and b by m. Similarly, for his formula (4), which we may rewrite as

$$F_{n+x}F_m - F_nF_{m+x} = (-1)^{m+1}F_{n-m}F_x\,,$$

we have only to set $x = a$, $m = n+b$ and use (3) again to get (2), and all steps are reversible. The reader may similarly derive the other formulas.

For reference to the history of (2), see [1, p. 404], [2], [3]. Formula (2) was posed as a problem [6]. Tagiuri is the oldest reference [4] of which I know. Formula (2) is the unifying theme behind all the formulas in [5].

[Continued on page 146.]