Substituting into (7) and changing the variable x to z by $x = (2/\sqrt{5})z$, obtain

$$(1 - z^2)T''_{2n+1}(z) - z \cdot T'_{2n+1}(z) + (2n+1)^2 T_{2n+1}(z) = 0$$

defining the required polynomials [4: 22.6.9 p. 781]. The case for k even may be handled similarly.

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[Continued from page 196.]

Let $k > 0$, $2 | k$, $K := 4k + 3$; the conditions

$$v_{k+1}r_k + v_k r_{k+1} = 2^K, \quad 0 < r_{k+1} \leqslant 2v_{k+1}, \quad 2 \nmid r_{k+1}$$

define the integers r_{k+1}, r_k uniquely. Then $2r_{k+1} < r_k$. Let

$$r_j := 2r_{j+1} + r_{j+2} \quad (j = k-1, k-2, \cdots, 1);$$

then

$$0 < 2r_{j+1} < r_j, \quad 2 \nmid r_j \Leftrightarrow 2 \nmid j, \quad v_{j+1}r_j + v_j r_{j+1} = 2^K \quad (j = k-1, k-2, \cdots, 1);$$

$j = 1$ gives

$$2r_1 + r_2 = 2^K, \quad 0 < 2r_1 < 2^K.$$

Let $y_k := 2 \cdot 2^K + r_1$, $x_k := 3y_k + 2^K$; then $2 \cdot 2^K < y_k$, $2 \nmid y_k$, $2 \nmid x_k$. The defining equation for x_k gives $H(x_k, y_k) = 2$. The defining equations for x_k, y_k, r_j $(j = 1, 2, \cdots, k-1)$ are the beginning of an algorithm by greatest and by nearest integers for x_k, y_k and therefore $N(x_k, y_k) > k$. For an arbitrary integer $s > 0$, let $g_s := x_s$, $h_s := y_s$ in case $2|s$ and $g_s := x_{s+1}$, $h_s := y_{s+1}$ in case $2 \nmid s$. This proves

Theorem 2. For every integer $s > 0$ there exist odd integers $g_s > h_s > 0$ with $E(g_s, h_s) \geqslant N(g_s, h_s) > s$, $H(g_s, h_s) = 2$.

Nothing is known about the average size of $H(a,b)$.

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