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EXPONENTIALS AND BESSEL FUNCTIONS

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A Bessell function of order n may be defined as follows:

$$J_n(x) = \sum_{\lambda=0}^{\infty} \frac{(-1)^\lambda}{\Gamma(\lambda+1)\Gamma(\lambda+n+1)} \left(\frac{x}{2}\right)^{n+2\lambda} \tag{1}$$

It may be easily shown that for integral $n, J_n(x)$ is the coefficient of U^n in the expansion of

$$\exp\left[\frac{x}{2}\left(u - \frac{1}{u}\right)\right]$$

i.e.,

$$\exp\left[\frac{x}{2}\left(u - \frac{1}{u}\right)\right] = \sum_{n=-\infty}^{\infty} U^n J_n(x) \tag{2}$$

Now let

$$u - \frac{1}{u} = L_{2k+1}, \tag{3}$$

where L_{2k+1} is a Lucas number defined by

$$L_1 = 1, \qquad L_2 = 3, \qquad L_n = L_{n-1} + L_{n-2}\quad, \tag{4}$$

where n is any integer.

Equation (3) becomes $u^2 - uL_{2k+1} - 1 = 0$ with roots

$$\left(\frac{1+\sqrt{5}}{2}\right)^{2k+1} = \alpha^{2k+1} \qquad \text{and} \qquad \left(\frac{1-\sqrt{5}}{2}\right)^{2k+1} = \beta^{2k+1},$$

where

$$\alpha = \frac{1+\sqrt{5}}{2} \qquad \text{and} \qquad \beta = \frac{1-\sqrt{5}}{2}$$

are the roots of the well known quadratic

$$\phi^2 = \phi + 1. \tag{5}$$

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