# THE GOLDEN SECTION AND THE ARTIST

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The readers of *The Fibonacci Quarterly,* interested for the most part in ramifications of their fascinating subject as expressed in mathematical terms, may also be interested in seeing what happens when the geometric harmonies inherent in the series are made visible to the eyes.

The ratio of the Fibonacci series, 1.618 or $\phi$, reciprocal 0.618, when drawn out rectangular form, produces the golden section rectangle (Fig. 1). The rectangle can be constructed geometrically by drawing a square, marking the center of the base and drawing a diagonal from this center to an opposite corner; then with this diagonal as a radius and the center base as center, drawing an arc that cuts a line extended from the base of the square. This will mark the end of a rectangle whose side will be in 1.618 ratio to the end. The end will be in 0.618 ratio to the side. The excess will itself be a $\phi$ rectangle.

A line parallel to the side through the point where the diagonal intersects the side of the square will mark off another $\phi$ rectangle with a square on its side in the excess, and a $\phi$ rectangle in the square; the remainder of the square will contain a $\phi$ rectangle with a square on its end.

Many instances of the presence of the golden section relation can be found in fine works of art preserved for their merits through the centuries. Some works of art can be found that have dimensions whose quotients are close to the ratio 1.618. In the cases studied, when these areas were subdivided geometrically as in Fig. 2, all main lines of the pictorial designs, and all minor directions and details were found to fall along lines of the diagram and diagonals to further subdivisions.

The subdivision of the $\phi$ rectangle can be accomplished geometrically by drawing lines parallel to the sides through the intersections of diagonals with the side of the square, and lines parallel to side and end through intersections of these lines with diagonals of square and excess, and through any other intersections that may occur (Fig. 2).

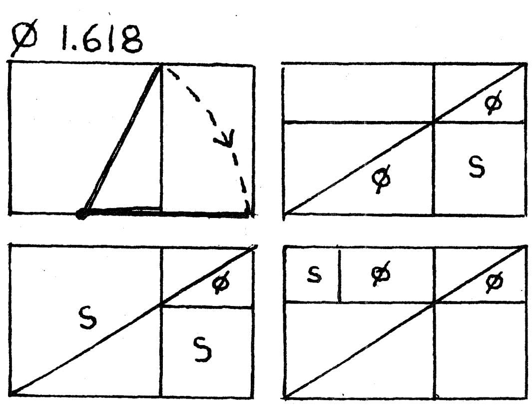



Figure 1

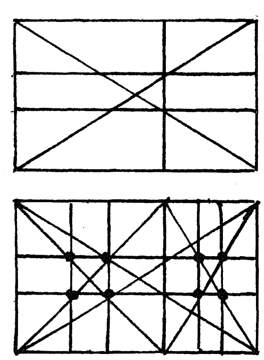


Figure 2

Or, it can be done perhaps more precisely by using the Fibonacci series.

The measurements of Giotto's *Ognissanti Madonna,* cl310, in the Uffizi, Fig. 3, fall just a little short of the 1.618 ratio. They are given as,

$$10'8'' \times 6'8'' = 128'' \times 80'' = 1.6$$

photos measure,

$$25.9 \times 16.1 \text{ cm} = 1.618 - .0093$$
$$13.4 \times 8.3 \text{ cm} = 1.618 - .0036$$

subdivision in the Fibonacci series:

| | | for practical purposes |
|---|---|---|
| $.618 \times 8.3 = 5.1294$ | 8.3 | 8.3 |
| | 5.1294 | 5.13 |
| | 3.1706 | 3.17 |
| | 1.9588 | 1.96 |
| | 1.2118 | 1.21 |
| | .747 | .75 |
| | .4648 | .46 |

When the golden section rectangle is applied to the photo of the painting, and the main divisions drawn, and the Fibonacci subdivisions are marked off on the edges, it will be found that the area occupied by the Madonna and Child lies precisely within a main $\phi$ division of the excess at the top, and a main $\phi$ division of the square at the bottom, and $\phi/2$ divisions at the sides. Architectural details, the angles of the steepled frame, vertical supports, centers of arcs, divisions of the platform, fall along subdivisions or along obliques from one subdivision to another. The lines of the top of the painting extend to center of golden section excess. The hands of the Madonna and Child, all lines of the angels, the tilt of their faces, their arms wings, the folds of their garments, fall along directions from one $\phi$ subdivision to another.

In making a study of the apparent incidence of certain geometric patterns in fine art, over 400 paintings of accepted excellence were analyzed. All but a few yielded to analysis. The majority clearly showed the presence of the $\phi$ relationship, or of its related shape, the $\sqrt{5}$ rectangle (Fig. 5). However, the overall shape of only a small number was in the simple 1.618 proportion. All followed the diagram lines in their designs. Among them we can mention: (Measurements starred are from photos of pictures shown within frames or borders, and are in centimeters. All others are dimensions given in catalogues or art histories, and are in inches.)

| | | |
|---|---|---|
| Duccio | *Madonna Enthroned* (Rucellai) 1285, Florence | $14.32 \times 8.85^* = 1.618$ |
| Duccio | *Madonna and Child,* Academy, Siena | $5.82 \times 3.6^* = 1.618$ |
| Martini | *Road to Calvary*, c1340 Louvre | $9\text{-}7/8 \times 6\text{-}1/8 = 1.618 - .0058$ |
| da Vinci | *Virgin of the Rocks,* 1483, Louvre | $78 \times 48 = 1.618 - .007$ |
| Turner | *Bay of Baise,* Tate Gal. | $57\frac{1}{2} \times 93\frac{1}{2} = 1.618 - .0008$ |
| Cole | *Florence from San Marco,* Cleveland Museum of Art, 1837 | $39 \times 63\text{-}1/8 = 1.618 - .0001$ |
| Romney | *Anne, Lady de la Pole,* 1786, MFA Boston | $95\frac{1}{2} \times 59 = 1.618 + .0006$ |

The photo of an Egyptian stele c. 2150 B.C., in the Metropolitan Museum of Art shows dimensions that have the 1.618 ratio. A seated figure fits exactly within the excess, heiroglyphic details fit in subdivisions of the square.

There is a bas-relief of an Assyrian winged demi-god of the 9th Century B.C. in the Metropolitan Museum of Art that fits perfectly into a 1.618 rectangle, and the strong lines of the wings, legs, beak follow divisions and diagonals of the $\phi$ diagram.

The Babylonian *Dying Lioness,* Ninevah, c. 600 B.C., in the British Museum, London, can also be contained exactly in a 1.618 rectangle. All lines of the figure, the directions of the arrows, fit on the lines of the diagram.

In a slab from the frieze of the Parthenon, c. 440 B.C., in the British Museum, showing two youths on prancing horses, the design also can be contained exactly in a $\phi$ rectangle and all lines conform to the pattern of the diagram.

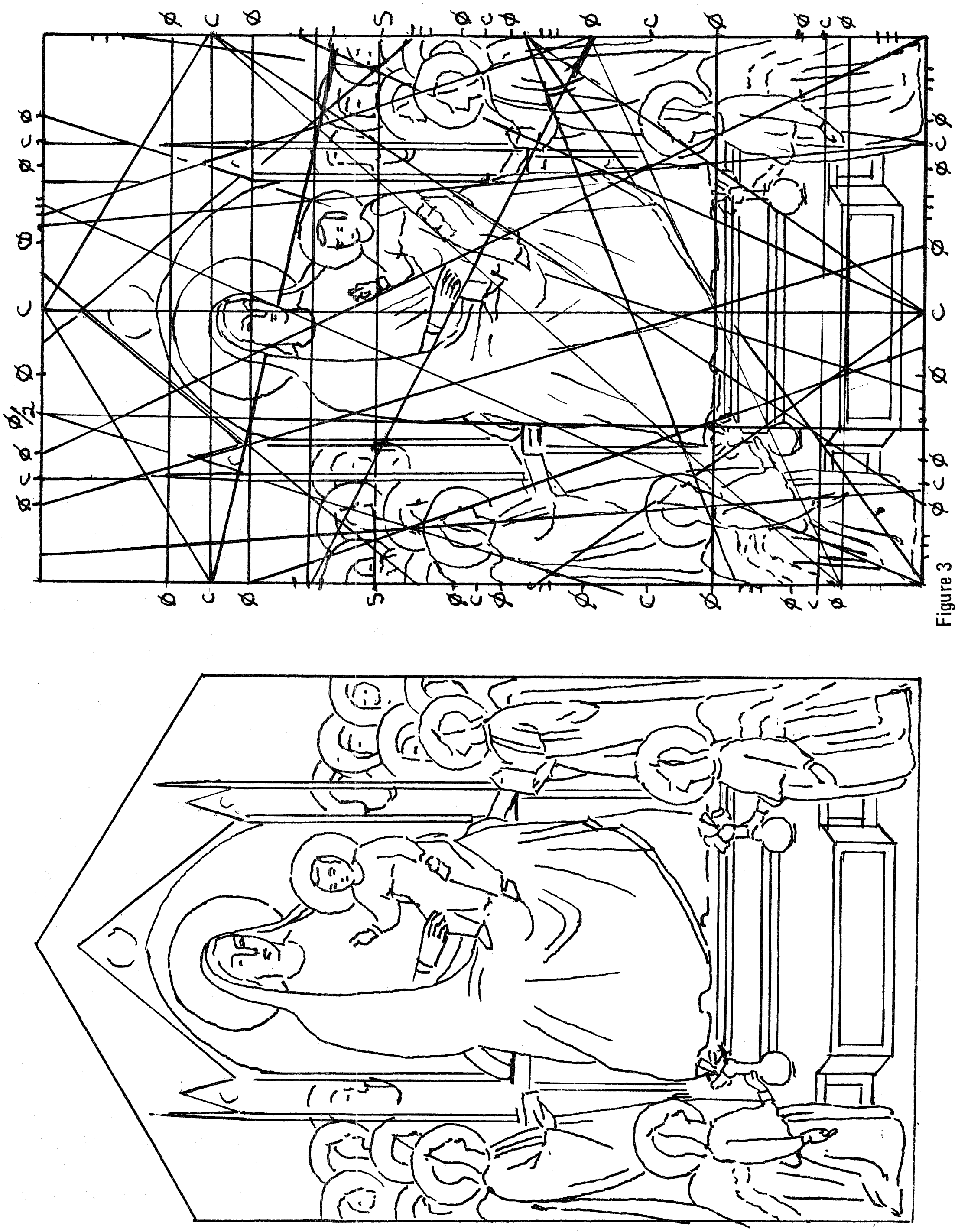


Figure 3

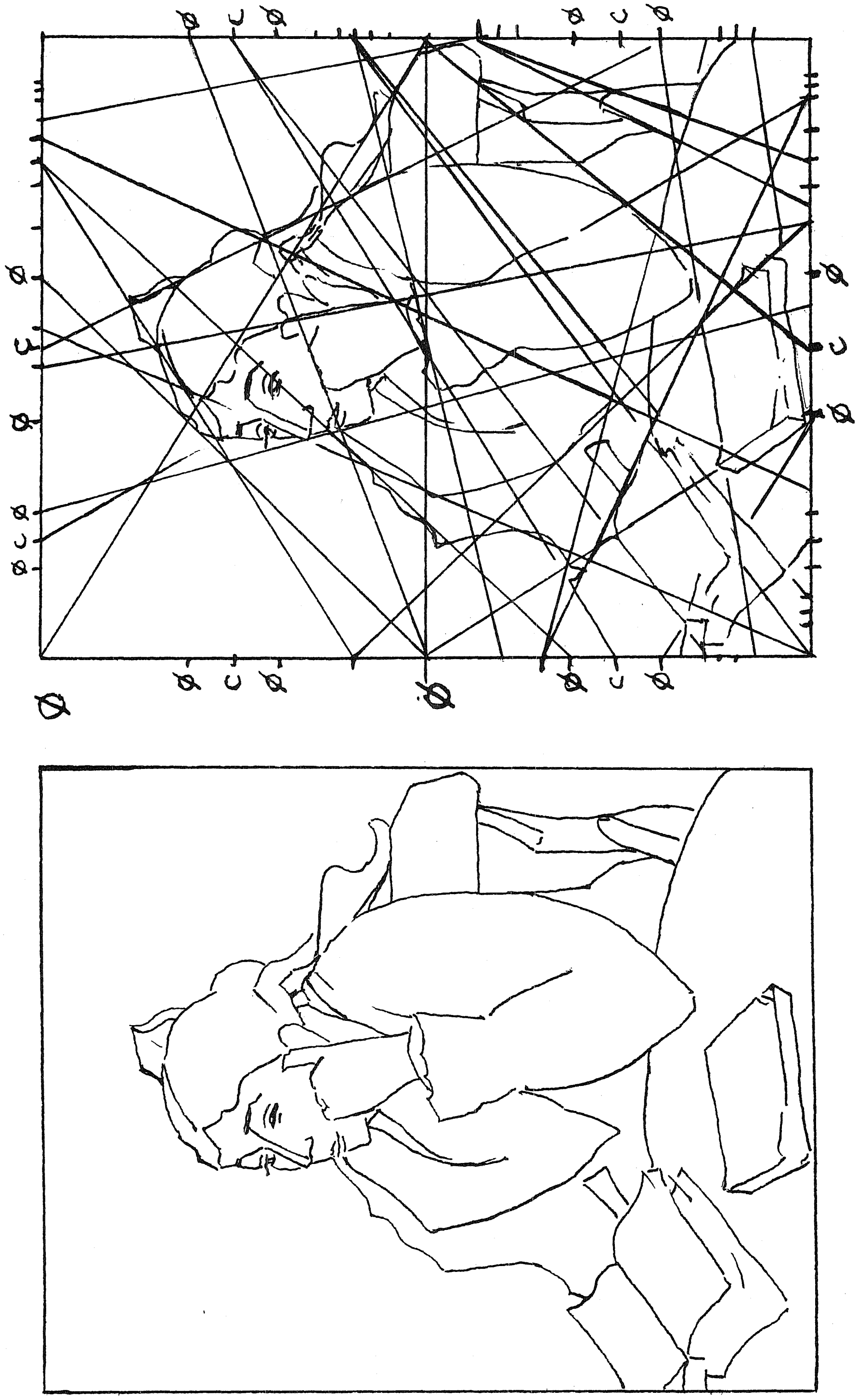
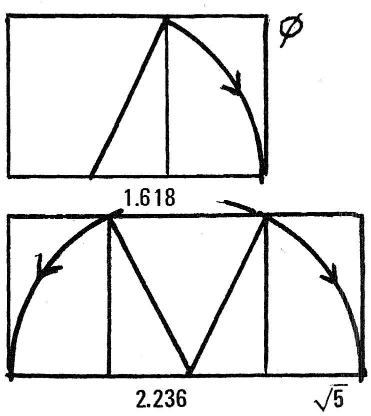


Figure 4



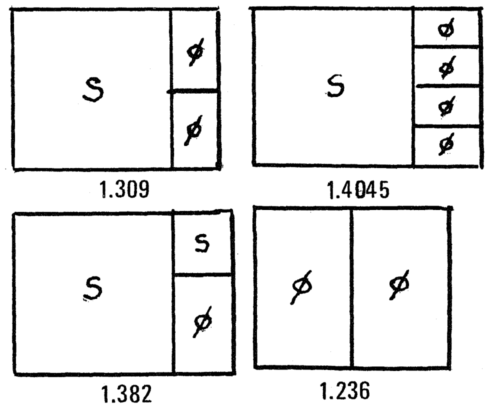




Figure 5 Figure 6

The measurements given for a marble balustrade relief in the Cathedral Baptistry, Civitale, Italy, c 725–750 A.D., are "about 3′ × 5′." In the photo, the border measures 8.2 × 13.25* = 1.618 – .001. All directions and details fit into the $\phi$ subdivisions.

The dimensions of many of the paintings studied yielded quotients close to the ratios of figures that consisted of sections of the $\phi$ rectangle, often combined with squares (Fig. 6):

$$1.309 = 1 + \frac{.618}{2}$$

$$1.4045 = 1 + \frac{1.618}{4}$$

$$1.302 = 1 + (1 - .618)$$

$$.809 = \frac{1.618}{2} \quad \text{(reciprocal, } 1.236 = 2 \times .618\text{)}.$$

We can see an example of one of these combined areas in *Yellow Accent,* 1947, private collection, by Jacques Villon (Fig. 7). The measurements of the photo of the picture shown in its frame are:

9.3 × 11.5* = 1.236 + .0004.

This couldn't be much closer to 1.236. To get subdivisions in the proportions of the Fibonacci series:

.618 × 9.3 = 5.7474

9.3
–5.7474
3.5526
2.1948
1.3578
.837
.5208
.316
.2046

When the edges of the painting are subdivided in these proportions, lines of the painting will be found to extend from one point of division to another precisely.

The same 1.236 framework can be found in *L'Arlesienne,* painted by Van Gogy in 1888. Its measurements are given as

36 × 29 = 1.236 + .0053
Photo 10.5 × 8.5* = 1.236 – .0007

All lines outlining areas and giving directions to details go from one $\phi$ division on the edge to another.

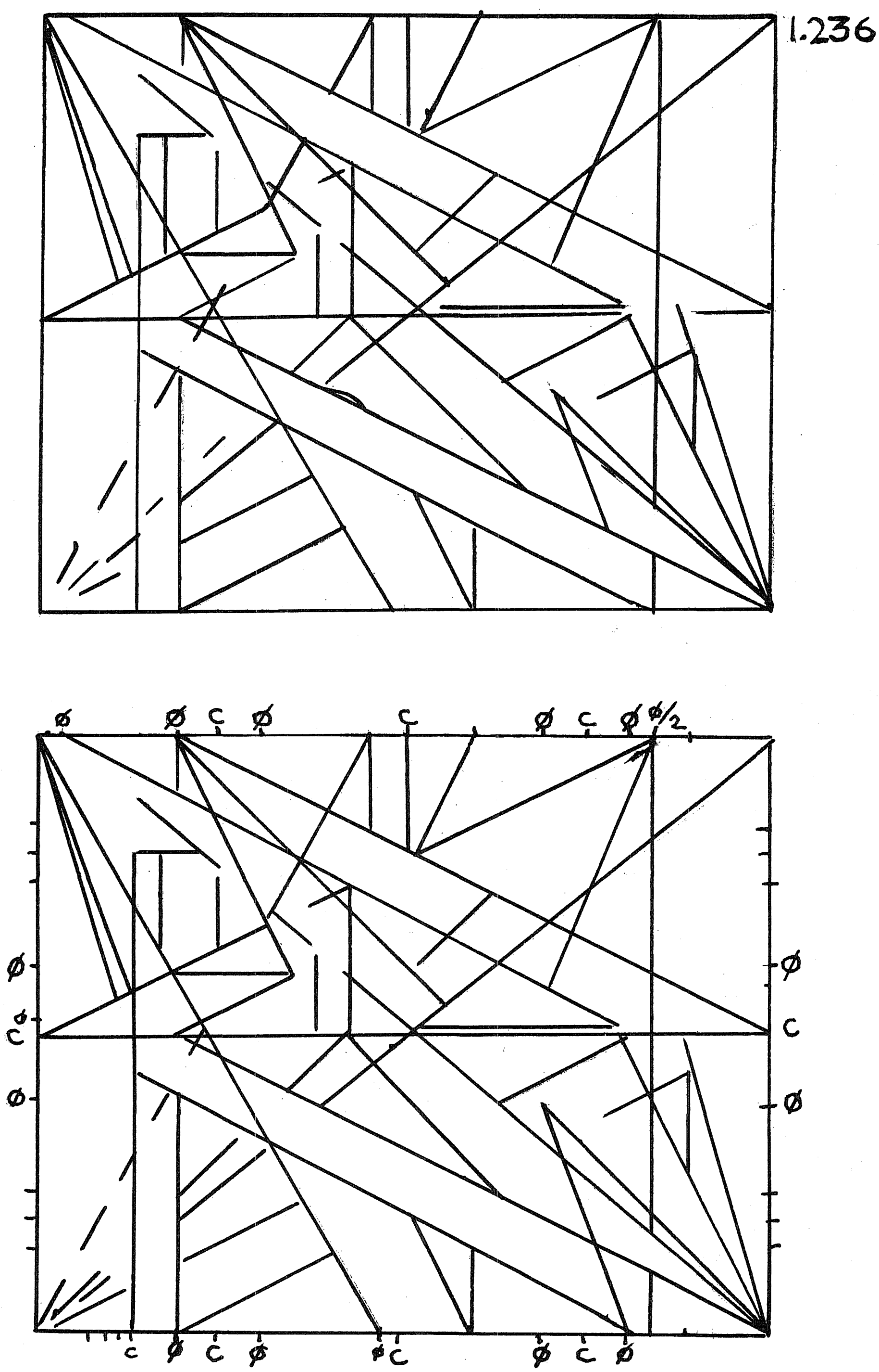



Figure 7

Among paintings that have ratios close to 1.236 and can be analyzed on that there are

| | | |
|---|---|---|
| Gos. Bk. of Ebbo | *St. Luke,* a. 823, Epernay | 5-3/8 × 6-7/8 = 1.236 − .0001 |
| Cloisters Apocalypse | *Opening of Book,* c 1320, Cloisters, N. Y. | 13.4 × 16.6* = 1.236 + .0028 |
| Cezanne | *Still Life,* c 1890, N G A Wash. | 25½ × 31½ = 1.236 − .0008 |
| Seurat | *Fishing Fleet,* c 1885, M Mod. A N Y | 8.85 × 10.9* = 1.236 − .0044 |
| Picasso | *Lady With Fan,* 1905, Harriman Col. | 39-3/4 × 32 = 1.236 − .0045 |
| Gris | *Painter's Window,* 1925, Baltimore M A | 39-1/4 × 31-3/4 = 1.236 − .0063 |

Many more complicated combinations were found. A figure made of a square plus an excess containing two $\sqrt{5}$ rectangles with a square on their side has the ratio 1.528 (Fig. 8).

An .809 shape with a $\phi$ rectangle across its side has the ratio 1.427.

Two $\sqrt{5}$ rectangles side-by-side has the ratio 1.118 (2.236/2).

All but a few paintings with dimensions that give quotients close to these ratios yielded to rigorous analysis.

The mathematical system on which this study was based was worked out in the early 1900's by Jay Hambidge, a minor American artist, who was interested in investigating several phases of art, particularly that of the classic Greek, in search of a possible mathematical basis for its apparent perfection. He measured hundreds of Greek vases in the Boston Museum of Art and the Metropolitan Museum in New York, and defined a series of figures basic to the combinations whose ratios kept recurring in the measurements of the vases. They were rectangles in the proportions of 1 to $\sqrt{2}$ (1.4142), $\sqrt{3}$ (1.732), $\sqrt{5}$ (2.236), and the golden rectangle, 1.618 or $\phi$.

To identify the various combinations that he found, and to properly subdivide them, he calculated their ratios and obtained their reciprocals. This mathematical material was not new, but his application of it to Greek art and his suggestion that artists should use it in their own work were new, and his clarification of the series of root rectangles, and their properties and interrelations evidently took even mathematicians by surprise.

He presented his discoveries in *Dynamic Symmetry: The Greek Vase* and *The Parthenon,* Yale University Press, 1920 and 1922. The general substance originally published in his review, *The Diagonal,* 1919–1920, and in *Elements of Dynamic Symmetry* is available now in a Dover publication, 1967.

In this study I have applied Hambidge's method of finding the specific geometric figure present in a work of art by identifying the quotient of its dimensions with the ratio of known geometric figures. As far as I know, this is an approach to the subject that has not been made before to works of art other than that of the Egyptians and Classic Greeks.

Hambidge thought that the system of planning works of art, vases, statues, murals, buildings, by the use of geometric frameworks disappeared with the classic Greeks, and that the Romans and others used what he called "static" symmetry, or a squared-off frame, which gave proportion in *line,* rather than in *area* (Fig 9).

However, it seems that evidences of the Greek knowledge of this process of geometric design can be found in later periods in many areas within the Greek sphere of influence. The first statues of Buddha were made in Gandhara in northwest India, which was settled by officers and soldiers from the remnants of Alexander's army and remained to some extent in contact with the western world.

There is a seated *Buddha,* c. 3rd Century A.D., in the Seattle Art Museum (Fig. 10), that shows the Greek influence in the treatment of hair and drapery. A $\phi$ rectangle can be applied to a front view photo of it, and all parts will be found to conform to the $\phi$ framework. This tradition seems to have persisted, as correlation with figures consisting of more complicated combinations of $\phi$ rectangles and squares can be found in a *Teaching Buddha* in Benares of the 5th Century A.D., and in an icon from South India, *Shiva* as *King of Dancers,* of the 12th Century A.D.

Other examples of works of art done in areas under Greek influence in which the $\phi$ rectangle or its combinations are apparent can be cited:

| | | | |
|---|---|---|---|
| | floor tiles | *Diana the Huntress* | square ÷ $\phi$ |
| | | *Still Life* | square ÷ $\phi$ |
| wall panels, | *Fish* | | 6.4 × 8.6* = 1.3455 − .0018 (Fig. 11) |
| | *Man and Lions* | | 7.9 × 5.9* = 1.3455 − .0015 |

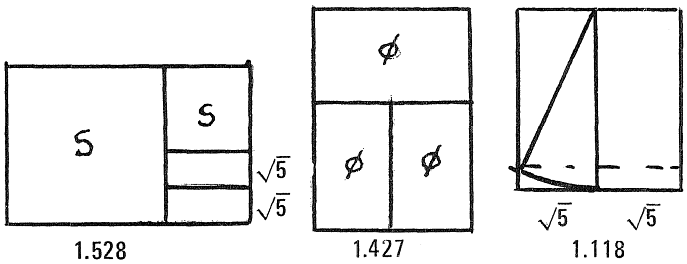



Figure 8

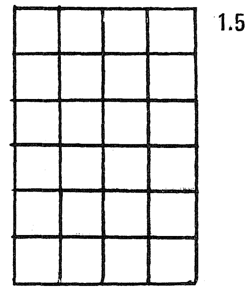



Figure 9

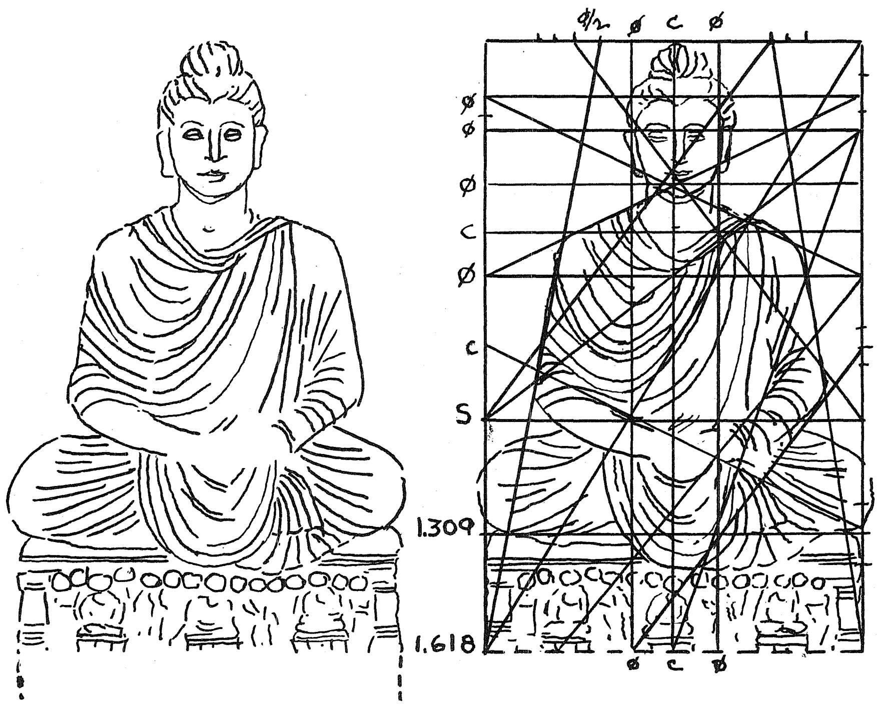



Figure 10

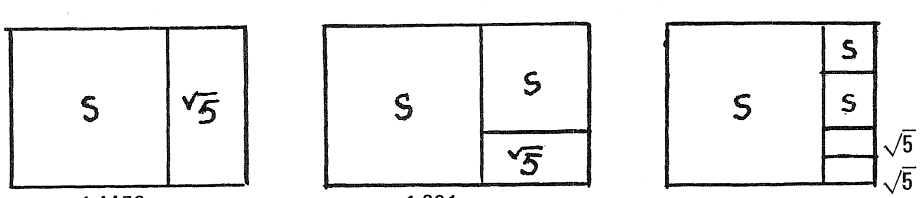



Figure 11

wall painting, *Hercules and Telephus* 9.9 × 8* = 1.236 + .0017
mms. *Georgics,* Bk. 111, 5th Century A.D., Vatican Library
*Shepherds Tending Flocks* 19 × 19.5* = 1.0225 + .0038 (.618 + .4045)

As the Graeco-Roman merges into the Early Christian culture, manuscript paintings, mosaics and frescoes still give evidence of the presence of geometric pattern on various $\phi$ arrangements, and now more frequently, on the $\sqrt{2}$ and $\sqrt{3}$ themes:

| | |
|---|---|
| Mosaics, 5th Century A.D., Santa Maria Maggiore, Rome | |
| *Abraham and Angels* | 19.8 × 17.2* = 1.1545 + .0046 |
| *Melchizedek and Abraham* | 19.3 × 14.8* = 1.309 + .0018 |
| Manuscripts | |
| Echternach Gospels, Ireland (?) c. 690 | |
| *Symbol of St. Mark* | 19 × 14.6* = 1.309 − .0008 |
| Book of Durrow, Irish, 7th Century, Trinity College, Dublin | |
| *Symbol of St. Matthew* | 6 × 13.9* = 2.309 + .0076 |
| Irish Gospel Book, St. Gall, 8th Century | |
| *St. Mark and Four Evangelists* | 19.7 × 14.65* = 1.3455 − .0008 |
| Registrum Gregoril, Trier, c. 985, Musee Conde, Chantilly | |
| *Emperor Otto II or III* | 20.8 × 15.4* = 1.3455 + .0051 |
| | given 10-5/8 × 7-7/8″ = 1.3455 + .0037 |
| Fresco, Catacomb of Commodilla, Rome, 7th Century | |
| *St. Luke* | 19 × 18 = 2 squares ÷ $\sqrt{2}$ |

All conform in their design to the geometric patterns indicated by the quotients of their dimensions.

The $\phi$ presence continues through the centuries unfolding into the Renaissance with the works of Duccio and Cimabue. Most of the paintings analyzed in this study fell within the Renaissance and Baroque periods, c. 1300 – c. 1660. Most of the artists were born in, or spent time in special areas, Venice, Florence, Milan, Umbria, Rome. One or another of these were also the dwelling places from time to time of the mathematicians Luca Paciola, Alberti, Bramanti, and the artist-mathematicians da Vinci, della Francesca, and Durer. The ratios found in this period included many combinations of the $\phi$ and $\sqrt{5}$ rectangles, of varying degrees of intricacy.

One of the combinations found is the 1.691 shape (Fig. 11). This consists of a square and an excess that contains a $\sqrt{5}$ rectangle with a square on its side. (Hambidge found this to be part of the floor plan of the Parthenon.) The ratio of the $\sqrt{5}$ rectangle is 2.236, its reciprocal is .4472. The ratio of the excess of the 1.691 figure will be 1.4472, reciprocal .691. Among works whose dimensions give a quotient close to 1.691, and yield to analysis are:

| | | |
|---|---|---|
| Rembrandt | *Goldweigher's Field* (etching) | 6.75 × 18.15* = 2.691 − .0022 |
| Sassetta | *Wolf of Gubbio* | 25.3 × 15* = 1.691 − .0044 |
| Sassetta | *St. Francis and the Bishop* | 26.09 × 15.4* = 1.691 |

If the excess of the 1.691 shape is divided in half longitudingly, the ratio of the square and this section will be

$$1 + \frac{.691}{2} = 1.3455$$

The excess will contain two squares and two $\sqrt{5}$ rectangles.

Among works whose dimensions yield quotients close to this figure and that analyze precisely are:

| | | |
|---|---|---|
| | *Avignon Pieta,* c. 1460, Louvre | 64 × 86 = 1.3455 − .0018 |
| Pollaiuolo (?) | *Portrait of Man,* Nat. Ga. Wash. | 20½ × 15¼ = 1.3455 − .0013 |
| Clouet | *Francis I,* Louvre c. 1525 | 28-1/8 × 32-3/4 = 1.3455 − .0033 |
| David | *Sabines,* 1799, Louvre | 152 × 204-3/4 = 1.3455 − .0011 |
| Beardsley | *Flosshilde* | 10.1 × 7.5* = 1.3455 + .0011 |

The *Isenheim Altarpiece,* 1511–1515, by Mather Grunewald, consists of a center panel, two side panels, and a base. Dimensions given are for the paintings within the frames, and are meaningless as geometric ratios. However, if the frames are included and the work is considered as a single plan, as sometimes happened in Medieval and Early Renaissance art, the overall dimensions measured on a photo of the complete work (Fig. 12), are:

$$26.55 \times 35.72^* = 1.3455.$$

The center panel plus the sides are contained in an area cut off by a $\phi$ division in the lower part of the square. Such are the interrelations of areas in the dynamic shapes that this area has the proportions

$$20.28 \times 35.72^* = 1.764\ (-.0022)(.764 = r\ 1.309).$$

The center panel, *The Crucifixion,* including the frame, is

$$20.28 \times 22.72^* = 1.118\ (-.0028).$$

The painting itself has strong lines of action, all of which coincide with divisions of the 1.118 shape or diagonals to prominent intersections.

The side panels, *St. Sebastian* and *St. Anthony* measure

$$17.5 \times 6.5^* = 2.691\ (+.0013).$$

The area remaining in the overall 1.3455 shape after the three panels are cut off consists of 2 $\phi$ rectangles, 2 squares, and a .4677 shape, reciprocal 2.1382 (the shape that Hambidge found to be the floor plan of the Parthenon). *The Entombment,* pictured on the stand, has areas and line directions that conform to subdivisions of the $\phi$ rectangles and squares in which they occur.

As far as I know, there is no concrete proof to show that the geometric relations found in the works of art were the result of deliberate planning on the part of the artists. The evidence is circumstantial.

There is a time pattern found in those examined. Pictorial designs on the $\sqrt{2}$ theme occurred c. 1200 – c. 1450, then seldom appeared again until the late 1800's. The $\phi$ theme was found throughout, peaking c. 1550, the $\sqrt{5}$ was most prevalent in the 1600's, the $\sqrt{\phi}$ in the 1700's, reappearing in the late 1800's.

There is the phenomenon of the irregularity of dimensions of paintings. Of the 400 studied, only about 1/8 had regular proportions, as 1-1/2, 1-1/3, etc. All the rest had odd measurements, as 70-1/2 × 53-1/2, 33 × 26, 18-1/2 × 16. The ratios of all could be closely related to ratios of geometric figures which were combinations of squares and $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ or $\phi$ rectangles. When the figures appropriate to the dimensions were applied to the paintings and properly subdivided, all lines of direction and demarcation of areas to smallest detail, fell into place on the parts of the diagram. The experience of finding this correlation tends to be very convincing to one who sees it happening over and over again.

Only a few clear clues were found. Fragments of dotted lines, vertical, horizontal, oblique, that fitted into a 1.472 shape, in background and design of a drawing by Poussin; an engraving by Durer in a 1.427 rectangle, a close copy by Raimondi in a 1.382 shape; construction lines of $\phi$ rectangles showing in the background of a 16th Century Japanese screen, whose panels had the ratios of 3.236 and 2.809.

Matila Ghyka, in his *Geometry in Art and Life,* has a chapter in which he presents evidence that a secret geometry based on the circle and pentagram was passed on from early Medieval times by secret ceremonies in the masons' guilds. He infers that a similar practice could have passed the knowledge down through the artists guilds. Ghyka shows instances of the $\phi$ rectangles in Renaissance art and architecture. He thought that knowledge of the system disappeared in the late 17th Century after van Dyke, and was rediscovered from time to time by individual artists, like Seurat, or by small cults.

However, instances of the presence of the $\phi$ rectangle, and of the special figure of the $\sqrt{\phi}$ (1.273) (Fig. 14) can be discerned in some 18th Century paintings, as,

| | | | | |
|---|---|---|---|---|
| Pater | *Bathera* | c1735 | Grenoble | 25-1/2 × 32-1/2 = 1.273 + .0015 |
| Boucher | *Bath of Diana* | 1742 | Louvre | 22-1/2 × 28-3/4 = 1.273 + .0047 |
| David | *Death of Marat* | 1793 | Brussells | 64 × 49 = 1.309 - .0029 |
| Watteau | *Gilles* | c1720 | Louvre | 58-3/8 × 72-1/4 = 1.235 - .0062 |
| Chardin | *Dessert* | 1741 | Louvre | 18-1/2 × 22 = 1.191 - .0018 |

All elements of the compositions relate closely to appropriate subdivisions. (.382)/2

Paintings by the early 19th Century artists working in the academic tradition also show $\phi$ relationships:

| | | | | |
|---|---|---|---|---|
| Ingres | *M. Bertin* | 1832 | Louvre | 37-1/2 × 46 = 1.236 - .009 |
| Delacroix | *Massacre at Scio* | 1824 | Louvre | 166 × 138-1/2 = 1.191 - .004 |
| Goya | *May 3, 1808* | 1814 | Prado | 104-3/4 × 135-7/8 = 1.309 - .0119 |
| Gericault | *Raft of the Medusa* | 1819 | Louvre | 193 × 282 = 1.4635 - .0024 (1.618 – .618/4) |

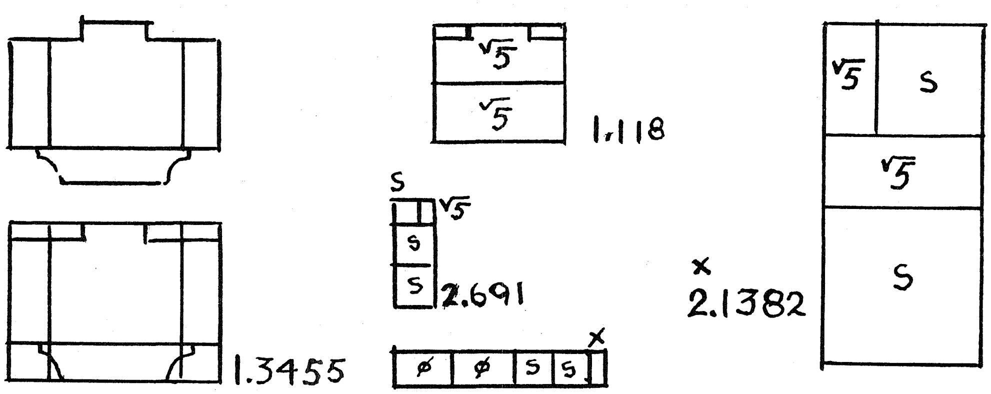



Figure 12

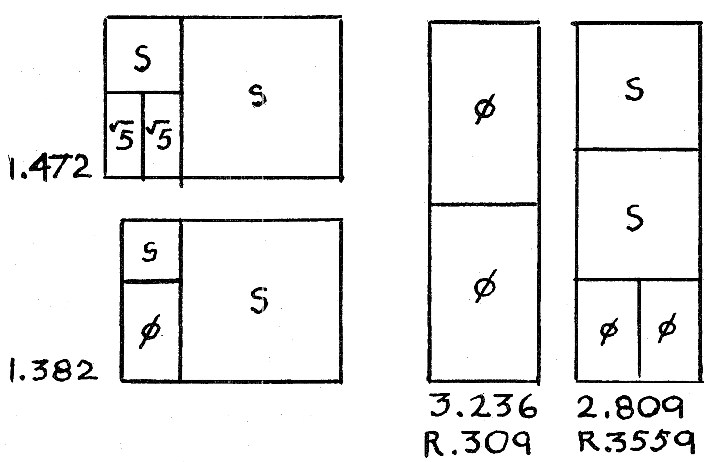



Figure 13

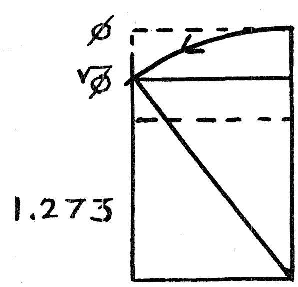



Figure 14

The upheaval in the art world, the split away from the academics that occurred in the middle of the 19th Century, is usually interpreted in terms of subject matter and technique. From my study, I am inclined to think that it was also partly related to the "liberation" of the knowledge of geometric design from the confines of the tight academic circle. Who was responsible for the disclosure? One of the Barbizon painters? Courbet? Someone made it known to the outsiders. From internal evidence, Manet had it, and Renoir, Degas, Toulouse-Lautrec, Cezanne, Seurat, Van Gogh.

In the late 80's, there was a group of artists led by Serusier, devoted to the study and application of the golden section. Bonnard and Vuillard were members of the group. They centered at Pont Avon, where Gauguin was in contact with them. His first well known painting, *Jacob Wrestling with the Angel,* was made there in 1888. It measures 28-3/4 × 36.5 = 1.273 − .0034 ($\sqrt{\phi}$), and analyzes perfectly on this pattern divided in $\phi$ ratio.

All of these artists were greatly interested in the newly revealed arts of Japan. We wonder to what extent they discerned the presence of geometric relationships in Japanese prints. These can be found clearly and definitely in the few examples of Japanese art examined. In four from a series *The Manga,* in the Metropolitan Museum, by Hokusai, 1817, the borders measure:

| | | | |
|---|---|---|---|
| | | | 10 × 14.45* = 1.4472 − .0022 (1 + $\sqrt{5}$) |
| sketches in style of Hokusai | | 1818 | 10.5 × 14.5* = 1.382 − .0022 |
| *Anecdotes* by Hokusai | | 1850 | 10.2 × 14.4* = 1.4142 − .0025 ($\sqrt{2}$) |
| *Red and White Peppers,* Freer Gallery | | 18th Century | 47 × 19 = 1.472 − .0017 |
| | | | 1 + (1.118 + 1) |
| *Horses,* Baltimore Museum of Art | | 17th Century | 4.5 × 12.2* = 2.7071 + .004 |
| | screen | | (.7071 = r 1.4142) |
| *Landscapes of the 4 Seasons* | | | 6.73 × 12.3* = 1.8284 − .0008 |
| | screen | | (.4142 + .4142) |
| *Han-Shan and Shih-te* | screen | 16th Century | 2.38 × 7.7* = 3.236 (r..309) |
| *Dai-itoku* M F A Boston, painting | | 11th Century | 6'3½" × 46½ = 1.618 + .005 |

All analyze precisely.

In Paris, about 1910 there was a group that called itself "Section d'Or," that investigated the use of this proportion. The group included Duchamp, Villon and Picabia. Matisse and Picasso were in contact with them:

| | | | |
|---|---|---|---|
| Duchamp | *Nude Descending Staircase* | 1912 | 58-3/8 × 35-3/8 = 1.644 + .006 |
| | | | (.809 + squares) |
| Villon | *Dinner Table* | 1912 | 27-3/4 × 32 = 1.236 + .0067 |
| Matisse | *Variation on de Heem* | 1915 | 71 × 87-3/4 = 1.236 − .0008 |
| Picasso | *Lady with a Fan* | 1905 | 39-3/4 × 32 = 1.236 − .0045 |

One wonders also how the revelation of the geometric system by the publication of Hambidge's investigations of Greek art, the probable original source, affected those who were in possession of the secret, who were still an "elect" group. At about the time of the revelation of Hambidge's discoveries, some artists in Paris, and Duchamp and Picabia in New York, started in a new direction, leading to Dada and Surrealism, the antithesis of the ideal of the order of Cubism and Dynamic Symmetry. This movement succeeded in the predominance of Surrealism in the 30's, which to some extent dampened interest in the order of geometric design.

The theory behind this study is that down through the ages from Classic Greek times, the knowledge of the process of geometric design was the possession of carefully chosen groups sworn to secrecy. That, of all art produced at any one time, their works are the ones that have mostly survived, partly because those chosen would naturally be the better artists, partly because of the superior effect the ordered proportions gave to their works.

What effect will the placing of this knowledge at the disposal of all artists have? Hambidge seemed to expect that artists would eagerly sieze upon his findings and use them in their work, and thus raise the quality of art on all levels. This did happen to a certain extent in illustration, advertising design and layout, industrial design, architecture and interior design, paralleling similar developments stemming from the Cubist movement in Europe. Hambidge was obviously unaware of the experiments with the golden section of Seurat, or of the Serusier group, or of the Section d'Or. Among outstanding American painters of the time who adopted the system we can mention Leon Kroll, George Bellows, Robert Henri and Jonas Lie.

Many others in the art field closed their eyes to the whole idea. If they should find it to be true, they would have to rethink all their concepts about art. Artists, art critics and historians often are not inclined to mathematics, and tend to shy away from it as something they don't know much about, and would have to make an effort to understand.

It takes some mental effort to understand and use the geometrical diagrams. Some can't do it; some, who must "paint as the bird sings" find it confusing to the point of interrupting their intuitive inspiration. Many artists resented the proposition that proportion and line direction, that they had worked so hard to master, could be achieved easily and perhaps more effectively by the use of a diagram. Many, not versed in mathematics, cannot appreciate the beauty of order in mathematics, and interpret it as "mechanical."

Will the situation resolve itself as before—the survival of the fittest—only now with the means of survival open to all those equal to grasping it? Or will the secret handed down through the ages as a "precious jewel" to those carefully selected for ability and responsibility, be diffused and list in indifference and sloth?

## SOURCES

Jay Hambidge, *Dynamic Symmetri The Greek Vase,* Yale University Press, 1920.
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From (5) it can be shown by induction that

(6) $$\alpha^n = \alpha F_n + F_{n-1} \quad \text{and} \quad \beta^n = \beta F_n + F_{n-1},$$

where $F_n$ and $F_{n-1}$ are Fibonacci numbers defined for integral $n$ by

(7) $$F_0 = 0, \quad F_1 = 1, \quad F_{n+1} = F_n + F_{n-1}.$$

From (2) and (3) we may write

(8) $$\exp \frac{x}{2} L_{2k+1} = \sum_{n=-\infty}^{\infty} U^n J_n(x)$$

From (6), we specialize

$$U^n = \alpha^{(2k+1)n} = \alpha F_{(2k+1)n} + F_{(2k+1)n-1}$$
$$U^n = \beta^{(2k+1)n} = \beta F_{(2k+1)n} + F_{(2k+1)n-1}.$$

Therefore (8) becomes

(9a) $$\exp\left(\frac{x}{2} L_{2k+1}\right) = \alpha \sum_{n=-\infty}^{\infty} F_{(2k+1)n} J_n(x) + \sum_{n=-\infty}^{\infty} F_{(2k+1)n-1} J_n(x)$$

and

(9b) $$\exp\left(\frac{x}{2} L_{2k+1}\right) = \beta \sum_{n=-\infty}^{\infty} F_{(2k+1)n} J_n(x) + \sum_{n=-\infty}^{\infty} F_{(2k+1)n-1} J_n(x)$$

[Continued on page 426.]