$$\lim_{N\to\infty} A = \frac{1}{F_k} - \frac{\sqrt{5}}{2} + \frac{5F_{2k}}{2L_k^2} = \frac{1}{F_k} - \frac{\sqrt{5}}{2} + \frac{5F_k}{2L_k}, \quad k \text{ odd.}$$

However, if we let k be even, then (2) gives us

$$L_k^2 = L_{2k}+2, \qquad L_k^2-4 = L_{2k}-2 = 5F_k^2,$$

so that our limit becomes

$$\lim_{N\to\infty} A = \frac{1}{F_k} - \frac{\sqrt{5}}{2} + \frac{5F_{2k}}{2(5F_k^2)} = \frac{1}{F_k} - \frac{\sqrt{5}}{2} + \frac{L_k}{2F_k}, \quad k \text{ even}$$

Finally,

$$\sum_{n=0}^{\infty} 1/F_{2^nk} = \begin{cases} \dfrac{2L_k - F_{2k}\sqrt{5} + 5F_k^2}{2F_{2k}}, & k \text{ odd;} \\ \dfrac{2 - F_k\sqrt{5} + L_k}{2F_k}, & k \text{ even.} \end{cases}$$

It would seem that the odd and even cases are closely related. First, let k be odd, or, $k = 2s+1$. Then

$$\sum_{n=0}^{\infty} 1/F_{(2s+1)2^n} = \frac{1}{F_{2s+1}} + \frac{5F_{2(2s+1)}}{2L_{2s+1}^2} - \frac{\sqrt{5}}{2} = B.$$

Now, let k be even. Let $k = 2(2s+1)$, making

$$\sum_{n=0}^{\infty} 1/F_{2(2s+1)2^n} = \frac{1}{F_{2(2s+1)}} + \frac{L_{2(2s+1)}}{2F_{2(2s+1)}} - \frac{\sqrt{5}}{2} = C.$$

Then, notice that $B = C + 1/F_{2s+1}$.

REFERENCES

1. I. J. Good, "A Reciprocal Series of Fibonacci Numbers," *The Fibonacci Quarterly,* Vol. 12, No. 4 (Dec., 1974), p. 346.
2. D. A. Millin, Problem H-237, *The Fibonacci Quarterly,* Vol. 12, No. 3 (Oct., 1974), p. 309.
3. Ken Siler, "Fibonacci Summations," *The Fibonacci Quarterly,* Vol. 1, No. 3 (Oct., 1963), pp. 67–69.
4. V. E. Hoggatt, Jr., and Marjorie Bicknell, "A Primer for the Fibonacci Numbers, Part XV: Variations on Summing a Series of Reciprocals of Fibonacci Numbers with Subscripts F_{2^nk}," *The Fibonacci Quarterly,* Vol. 14, No. 3 (Oct. 1976), pp. 272–276.

****

[Cont. from p. 452.]

RESPONSE

We push Pascal to the left, up tight,
To see what else can be brought to light.
In flowers and trees the world around,
The Fibonacci numbers do abound.
Look up to the right while taking sums.
What you find there will strike you dumb.

. . . Verner E. Hoggatt, Jr.
San Jose State University
San Jose, CA 95192