Corollary 3. The only periods that a P.F.L.S. $\{J_{a,1}\}$ can have is 2 or $p-1$, the only even numbers dividing $p-1$. It is easily seen that $\frac{1}{2}(p-3)$ of these P.F.L.S. have a period of $p-1$, each giving rise to one Fibonacci-like group with a period of $\frac{1}{2}(p-1)$ and one with a period of $p-1$. Those with periods of $\frac{1}{2}(p-1)$ correspond to the quadratic residues of p excluding 1, and the others correspond to the quadratic non-residues, excluding −1.

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SOLUTION OF A CERTAIN RECURRENCE RELATION

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At the recent research conference of the Fibonacci Association, Marjorie Bicknell-Johnson gave the recurrence relation

$$P_{r+1} - 2P_r - P_{r-1} + P_{r-2} = 0, \qquad r = 3, 4, \cdots, \tag{1}$$

that represents the number of paths for r reflections in three glass plates (with initial values $P_1 = 1$, $P_2 = 3$ and $P_3 = 6$). I submit here an explicit expression for P_r, and also obtain its generating function.

Based on the usual theory for such relationships, the general solution of (1) can be given in the form

$$P_r = C_1R_1^r + C_2R_2^r + C_3R_3^r, \tag{2}$$

where the quantities R_1, R_2 and R_3 are the roots of the equation

$$R^3 - 2R^2 - R + 1 = 0, \tag{3}$$

and the constants C_1, C_2 and C_3 must be determined to fit the specified conditions.

This cubic, whose discriminant is equal to 49, has three real roots, and they can best be expressed in trigonometric form, as texts on theory of equations seem to say. The roots of (3) are

$$\begin{cases} R_1 = \frac{2}{3}[1 + \sqrt{7}\cos\phi] \\ R_2 = \frac{1}{3}[2 - \sqrt{7}\cos\phi + \sqrt{21}\sin\phi] \\ R_3 = \frac{1}{3}[2 - \sqrt{7}\cos\phi - \sqrt{21}\sin\phi] \end{cases}, \tag{4}$$

where

$$\phi = \frac{1}{3}\arccos\left(\frac{1}{2\sqrt{7}}\right). \tag{5}$$

Such roots can be represented exactly only if they are left in this form. (Approximations of them are

$$R_1 = 2.2469796, \qquad R_2 = 0.5549581, \quad \text{and} \quad R_3 = -0.8019377.)$$

The constants in the solution (2) are then found by solving the linear system

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