

$$P.2. \quad \sum_{r=0}^n |{}^n T_r^*| = |{}^{n+1} T_0^*| \quad (= 1 \text{ for } n = 0).$$

REMARKS. We wish to draw attention to the fact that we obtained Tribonacci Numbers from Stanton and Cowan's Diagram. Such a generalization to higher dimensions may be possible but it is very complicated as it is exceedingly difficult to picture these numbers. However there are other ways of obtaining these numbers as for example Tribonacci numbers from the expansion of $(1+x+x^2)^n$ [4].

REFERENCES

1. R. G. Stanton and D. D. Gowan, "Note on a Square Functional Equation," *SIAM Rev.*, 12, 1970, pp. 277-279.
2. L. Carlitz, "Some q Analogues of Certain Combinatorial Numbers," *SIAM, Math. Anal.*, 1973, 4, pp. 433-446.
3. K. Alladi, "On Fibonacci Polynomials and Their Generalization," *The Fibonacci Quarterly*, to appear.
4. V. E. Hoggatt, Jr., and Marjorie Bicknell, "Generalized Fibonacci Polynomials," *The Fibonacci Quarterly*, Vol. 11, No. 5 (1973), p. 457.

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$$(6) \quad \begin{cases} C_1 R_1 + C_2 R_2 + C_3 R_3 = 1 \\ C_1 R_1^2 + C_2 R_2^2 + C_3 R_3^2 = 3 \\ C_1 R_1^3 + C_2 R_2^3 + C_3 R_3^3 = 6, \end{cases}$$

whose determinant is

$$(7) \quad D = \begin{vmatrix} R_1 & R_2 & R_3 \\ R_1^2 & R_2^2 & R_3^2 \\ R_1^3 & R_2^3 & R_3^3 \end{vmatrix} = (R_1 - R_2)(R_1 - R_3)(R_2 - R_3) = 7.$$

Thus, using Cramer's rule, one obtains constants as

$$(8) \quad \begin{cases} C_1 = \frac{1}{7} R_2 R_3 (R_3 - R_2) [6 - 3(R_2 + R_3) + R_2 R_3] \\ C_2 = \frac{1}{7} R_1 R_3 (R_1 - R_3) [6 - 3(R_1 + R_3) + R_1 R_3] \\ C_3 = \frac{1}{7} R_1 R_2 (R_2 - R_1) [6 - 3(R_1 + R_2) + R_1 R_2], \end{cases}$$

which reduce simply to the fixed numbers

$$(9) \quad C_1 = \frac{1}{7} (3 - R_3), \quad C_2 = \frac{1}{7} (3 - R_1), \quad C_3 = \frac{1}{7} (3 - R_2)$$

when many discovered relations between the three roots are taken into account. These involve the following.

Relations between the roots and the coefficient of the cubic gives

$$(10) \quad R_1 + R_2 + R_3 = 2, \quad R_1 R_2 + R_1 R_3 + R_2 R_3 = -1, \quad R_1 R_2 R_3 = -1,$$

while from the discriminant we have

$$(11) \quad (R_1 - R_2)(R_1 - R_3)(R_2 - R_3) = \sqrt{49} = 7.$$

Use of these and the relation $R_1^2 + R_2^2 + R_3^2 = 6$ furnish, after some manipulation,

$$(12) \quad \begin{cases} R_1 R_3^2 + R_2 R_1^2 + R_3 R_2^2 = 4 \\ R_1 R_2^2 + R_2 R_3^2 + R_3 R_1^2 = -3. \end{cases}$$

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