$$F_{m,1}(u,v) = 2u^{m-1} + (m-1)u^m \qquad (m \geq 1),$$

so that

$$F_{1,n}(u,v) = 2v^{n-1} + (n-1)v^n \qquad (n \geq 1).$$

For $m = n$ we get

$$(6.10) \qquad F_{m,m}(u,v) = 2\sum_{r=0}^{m-1} \binom{m-1}{r}^2 (uv)^r + (u+v)\sum_{r=0}^{m-1} \binom{m-1}{r}\binom{m-1}{r+1}(uv)^r .$$

In connection with the recurrence (6.4), it may be of interest to point out that Stanton and Cowan [3] have discussed the recurrence

$$(6.11) \qquad g(n+1, r+1) = g(n, r+1) + g(n+1, r) + g(n, r)$$

subject to the initial conditions

$$g(n,0) = g(0,r) = 1 \qquad (n \geq 0,\ r \geq 0).$$

The more general recurrences

$$(6.12) \qquad A(n,r) = A(n-1, r-1) + q^n A(n, r-1) + q^r A(n-1, r)$$

and

$$(6.13) \qquad A(n, r) = A(n-1, r-1) + p^n A(n, r-1) + q^r A(n-1, r)$$

have been treated in [2].

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1. L. Carlitz, "Fibonacci Notes. 1. Zero-one Sequences and Fibonacci Numbers of Higher Order," *The Fibonacci Quarterly,* Vol. 12 (1974), pp. 1–10.
2. L. Carlitz, "Some q-analogs of Certain Combinatorial Numbers," *SIAM J. on Math. Analysis,* Vol. 4 (1973), pp. 433–446.
3. R. G. Stanton and D. D. Cowan, "Note on a 'Square' Functional Equation," *SIAM Review,* Vol. 12 (1972), pp. 277–279.

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[Continued from page 45.]

If the relations (10), (11) and (12) are used, it can be shown that the much simpler expressions for the constants in the explicit solution (2) are indeed given by equations (9).

The generating function for the sequence P_r is defined by

$$(13) \qquad G = \sum_{r=0}^{\infty} x^r P_r = \sum_{r=0}^{\infty} [C_1(xR_1)^r + C_2(xR_2)^r + C_3(xR_3)^r],$$

If we now make use of the summation of a geometric series, then

$$(14) \qquad G = \frac{C_1}{1-xR_1} + \frac{C_2}{1-xR_2} + \frac{C_3}{1-xR_3}$$

$$= \frac{C_1(1-xR_2)(1-xR_3) + C_2(1-xR_1)(1-xR_3) + C_3(1-xR_1)(1-xR_2)}{1 - x(R_1+R_2+R_3) + x^2(R_1R_2 + R_1R_3 + R_2R_3) - x^3R_1R_2R_3},$$

which, upon employing the relations (9), (10), (11) and (12), finally reduces to the simple equation

$$(15) \qquad G = \frac{1-x}{1-2x-x^2+x^3} .$$

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