$$ST(N_{k+1})\cdot ST(N_{j+1}).$$

Thus

$$ST(W_{k,1}) = L_{2k+2} - 2 + 2F_{2k} + 2(F_2 + F_4 + \cdots + F_{2k-2}),$$

and, if $j \geq 2$,

$$ST(W_{k,j}) = L_{2k+2j} - 2 + 2F_{2k}F_{2j} + 2F_{2j}(F_2 + F_4 + \cdots + F_{2k-2}) + 2F_{2k}(F_2 + F_4 + \cdots + F_{2j-2}).$$

Simple Fibonacci identities reduce these equations to the desired formula.

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THE DIOPHANTINE EQUATION $(x_1 + x_2 + \cdots + x_n)^2 = x_1^3 + x_2^3 + \cdots + x_n^3$

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The Diophantine equation

$$(1)\qquad (x_1 + x_2 + \cdots + x_n)^2 = x_1^3 + x_2^3 + \cdots + x_n^3$$

has the non-trivial solution $x_i = i$ as well as permutations of this n-tuple since

$$\sum_{i=1}^{n} i = n(n+1)/2 \quad \text{and} \quad \sum_{i=1}^{n} i^3 = n^2(n+1)^2/4.$$

Also, for any n, $x_i = n$ for all $i = 1, 2, \cdots, n$, is a solution of (1). Thus, (1) has an infinite number of non-trivial solutions in positive integers.

On the other hand if one assumes $x_i > 0$, then for each i one has $x_i < n^2$. To see this, let a be the largest coordinate in a solution $(x_1, x_2, \cdots, x_n)$. Then,

$$x_1 + x_2 + \cdots + x_n \leq na.$$

For the same solution

$$x_1^3 + x_2^3 + \cdots + x_n^3 \geq a^3$$

and so $a \leq n^2$. Thus, we see that for a fixed positive integer, n, equation (1) has only a finite number of solutions in positive integers and we have proved the following theorem.

Theorem. Equation (1) has only a finite number of solutions in positive integers for a fixed positive integer n but as $n \to \infty$ the number of solutions is unbounded.

Clearly if $(x_1, x_2, \cdots, x_n)$ is a solution of (1) wherein some entry is zero, then one has knowledge of a solution (1) for $n - 1$ and so, except for $n = 1$, we exclude all solutions with a zero coordinate hereafter.

[Continued on page 16.]