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BINET'S FORMULA GENERALIZED

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Any generalization of the Fibonacci sequence $\{F_n\} = 1, 1, 2, 3, 5, 8, 13, 21, \cdots$ necessarily involves a change in one or both of the defining equations

(1) $$F_1 = F_2 = 1, \qquad F_{n+2} = F_{n+1} + F_n \qquad (n \geqslant 1).$$

Here, however, we seek such a generalization indirectly, by starting with *Binet's formula*

$$F_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}} \qquad (n \geqslant 1)$$

instead of (1). Suppose we define, for any positive integer p, the sequence G_n by

(2) $$G_n = \frac{\left(\frac{1+\sqrt{p}}{2}\right)^n - \left(\frac{1-\sqrt{p}}{2}\right)^n}{\sqrt{p}} \qquad (n \geqslant 1).$$

Thus $\{G_n\} = \{F_n\}$ in the case $p = 5$. We can also write

(3) $$G_n = \frac{\alpha^n - \beta^n}{\sqrt{p}} \qquad (n \geqslant 1),$$

where

$$\alpha = \frac{1+\sqrt{p}}{2}, \qquad \beta = \frac{1-\sqrt{p}}{2}$$

are roots of the equation

(4) $$x^2 - x - \left(\frac{p-1}{4}\right) = 0.$$

Corresponding to (1), we now have the equations

(5) $$G_1 = G_2 = 1, \qquad G_{n+2} = G_{n+1} + \left(\frac{p-1}{4}\right) G_n \qquad (n \geqslant 1).$$

Proof. Clearly $\alpha - \beta = \sqrt{p}$ and $\alpha + \beta = 1$, so that (3) implies

$$G_1 = \frac{\alpha-\beta}{\sqrt{p}} = 1, \qquad G_2 = \frac{(\alpha-\beta)(\alpha+\beta)}{\sqrt{p}} = 1.$$

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