9. Problem B-74, Posed by M. N. S. Swamy, *The Fibonacci Quarterly,* Vol. 3, No. 3 (Oct., 1965), p. 236; Solved by D. Zeitlin, *ibid.,* Vol. 4, No. 1 (Feb. 1966), pp. 94–96.
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[Continued from page 24.]

(iii) $$\left(\frac{p-1}{4}\right) G_n^2 + G_{n+1}^2 = G_{2n+1} \qquad (n \geqslant 1)$$

(iv) $$G_{n+2}^2 - \left(\frac{p-1}{4}\right)^2 G_n^2 = G_{2n+2} \qquad (n \geqslant 1)$$

(v) $$G_n = \sum_{r=0}^{n-1} \binom{n-1-r}{r} \left(\frac{p-1}{4}\right)^r \qquad (n \geqslant 1)$$

(vi) $$\left(\frac{p-1}{4}\right) \sum_{r=1}^{n} G_r = G_{n+2} - 1 \qquad (n \geqslant 1).$$

The proofs of the above results, which rely essentially on equations (2), (3) and (5), together with

$$\alpha - \beta = \sqrt{p}, \qquad \alpha + \beta = 1 \quad \text{and} \quad \alpha\beta = -\left(\frac{p-1}{4}\right),$$

are fairly straightforward and left to the reader. Of course, results such as these are not new. For example, (ii) was proved in a slightly more general form by E. Lucas as early as 1876 (see [1] page 396).

Finally, turning to the *vertical* sequences in the table given earlier, it follows from (v) that the sequence under G_n $(n \geqslant 1)$ is given by

(6) $$\left\{ \sum_{r=0}^{n-1} \binom{n-1-r}{r} (k-1)^r \right\} \qquad (k \geqslant 1),$$

so that for example the sequences under G_4 and G_5 are $\{2k-1\}$ and $\{k^2+k-1\}$, respectively. Alternatively, instead of using (6), we can apply the Binomial Theorem to (2) and obtain the general vertical sequence in the form

$$\left\{ \frac{1}{2^{n-1}} \sum_{\substack{r=1 \\ r \text{ odd}}}^{n} \binom{n}{r} (4k-3)^{(r-1)/2} \right\} \qquad (k \geqslant 1).$$

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1. L. E. Dickson, *History of the Theory of Numbers,* Vol. 1, Carnegie Institution (Washington 1919).

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