(29) $$\lim_{i \to \infty} \frac{g_{i+k}^{(n)}}{g_{i+m}^{(n+1)}} = 0.$$

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SUMMATION OF MULTIPARAMETER HARMONIC SERIES

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1. INTRODUCTION

Consider the multiparameter alternating harmonic series denoted and defined by

(1) $$\omega(j; k_1, \cdots, k_n) = \sum_{i=0}^{\infty} (-1)^i/(j+s_i),$$

where j and the k_i are positive integers, $s_0 = 0$, $s_n = S$, and

$$s_i = [i/n]S + \sum_{t=1}^{i, \mathrm{mod}\, n} k_t .$$

Note that the parameters $k_1, \cdots, k_n$ are successive cyclic denominator increments. In the ensuing treatment summation formulas for such series, to be called ω-series, are developed which admit evaluation in terms of elementary functions. An example is included to illustrate the formulas.

2. SUMMATION FORMULAS

The expression of the summation formulas for the ω-series (1) is based upon the following two lemmas.

Lemma 1.

(2) $$\omega(j; k) = (\tfrac{1}{2}k)G(j/k) = \int_0^1 x^{j-1}dx/(1+x^k)$$

$$= (-1)^{j-1}(r/k)\ln(1+x)$$

$$-(2/k)\sum_{i=0}^{q-1} [P_i(x)\cos((2i+1)j\pi/k) - Q_i(x)\sin((2i+1)j\pi/k)]\Big|_0^1 ,$$

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