Note that no residue in Group 1 occurs in Group 2 but that corresponding residues in the two groups add up to $M = 200$. Also, these numbers have other unusual characteristics. Add any two and the sum will be some one or the other of the numbers or, if the sum is greater than 200, subtract $M = 200$ and the remainder will be found somewhere in the list. Subtract any two numbers with the same result. Of course, the reader's inspection has already noted that the numbers above the central 50 are arranged as mirror images of those below.

It is interesting to note that mirror-image molecules (stereoisomers) are of the utmost importance in biochemical considerations and in heredity. Since the connection between the Fibonacci Series and certain facts in heredity has long been noted, perhaps further investigation of the self-reproductive nature of the Fibonacci Series and of its tendency to form mirror images would be fruitful.

REFERENCES

1. Joseph Mandelson, "Amateur Interests in the Fibonacci Series III, Moduli and Residues," *The Fibonacci Quarterly,* Vol. 6, No. 4, October, 1968, pp. 275–278.
2. Brother Alfred Brousseau, "Entry Points and Periods for the Fibonacci Sequence," *Fibonacci and Related Number Theoretic Tables,* The Fibonacci Association, 1972.

****

[Continued from page 144.]

Proof. By pairwise association and use of the relationship [3, p. 285],

$$\psi'(x/S) = \sum_{i=0}^{\infty} 1/(i+x/S)^2$$

which is uniformly convergent for $x \geqslant 1$, one establishes

$$\omega(j;k_1,k_2) = \sum_{i=0}^{\infty} \int_j^{j+k_1} dx/(x+iS)^2$$

$$= (1/S)^2 \int_j^{j+k_1} \psi'(x/S)dx$$

which integrates into the statement (3). The integral form of the psi function occurring in (4) is listed in [4, p. 16] and the integral evaluation is a celebrated theorem of Gauss [3, p. 286; 4, p. 18].

Corollary. Formula (4) can be extended to an arbitrary positive rational argument via the identity [4, p. 16],

$$\psi(n+z) = \psi(z) + \sum_{i=0}^{n-1} 1/(z+i).$$

An ω-series with an arbitrary even number of k_i parameters can be grouped into a series of successive cycles of parametric incrementation within which the terms are pairwise associated. This procedure leads to an expression in terms of the biparameter ω-series, and application of Lemma 2 yields an explicit summation formula in terms of the psi function.

Theorem 1.

$$\omega(j;k_1,\cdots,k_{2n}) = \sum_{i=0}^{n-1} \omega(j+s_{2i};k_{2i+1},S-k_{2i+1})$$

$$= (1/S)\sum_{i=0}^{2n-1} (-1)^{i+1}\psi((j+s_{2i})/S).$$

[Continued on page 172.]