(3.8) $$F_n^{(2)} = [(5n^2-2)F_n - 3nL_n]/50$$

as well as (1.13). The algebra, however, is horrendous. The identity (3.8) can be derived by solving for the constants $A, B, C, D, E,$ and F in

$$F_n^{(2)} = (A+Bn+Cn^2)F_n + (D+En+Fn^2)L_n$$

which arises since $\{F_n^{(2)}\}$ has auxiliary polynomial $(x^2-x-1)^3$, whose roots are α, α, α and β, β, β.

Two other determinant identities follow without proof.

$$\begin{vmatrix} F_{n+2}^{(1)} & F_{n+1}^{(1)} & F_{n-1}^{(1)} \\ F_{n+1}^{(1)} & F_n^{(1)} & F_{n-2}^{(1)} \\ F_n^{(1)} & F_{n-1}^{(1)} & F_{n-3}^{(1)} \end{vmatrix} = (-1)^n[F_{n-5}^{(1)} + 2F_{n-4}]$$

$$\begin{vmatrix} F_{n+2}^{(1)} & F_n^{(1)} & F_{n-1}^{(1)} \\ F_{n+1}^{(1)} & F_{n-1}^{(1)} & F_{n-2}^{(1)} \\ F_n^{(1)} & F_{n-2}^{(1)} & F_{n-3}^{(1)} \end{vmatrix} = (-1)^n[F_{n-2}^{(1)} - F_{n-2}]$$

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TWO RECURSION RELATIONS FOR $F(F(n))$

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Some time ago, in [1], the question of the existence of a recursion relation for the sequence of Fibonacci numbers with Fibonacci numbers for subscripts was raised. In the present article we give a 6^{th} order non-linear recursion for $f(n) = F(F(n))$.

Proposition. Let $f(n) = F(F(n))$, where $F(n)$ is the n^{th} Fibonacci number, then

$$f(n) = (5f(n-2)^2 + (-1)^{F(n+1)})f(n-3) + (-1)^{F(n)}(f(n-3) - (-1)^{F(n+1)}f(n-6))f(n-2)/f(n-5).$$

Remark. Identity (1) below is given in [2], and identity (2) is proved similarly. Note also that $a \equiv b$ (mod 3) implies that

$$(-1)^{F(a)} = (-1)^{F(b)} = (-1)^{L(a)} = (-1)^{L(b)},$$

which is used frequently.

(1) $$F(a+b) = F(a)L(b) - (-1)^b F(a-b)$$

(2) $$5F(a)F(b) = L(a+b) - (-1)^a L(b-a).$$

Proof of Proposition. In (1), let $a = F(n-2)$, $b = F(n-1)$ to obtain

$$\begin{aligned} f(n) &= f(n-2)L(F(n-1)) - (-1)^{F(n-1)}F(-F(n-3)) \\ &= f(n-2)L(F(n-1)) - (-1)^{F(n-1)}(-1)^{F(n-3)+1}f(n-3) \\ &= f(n-2)L(F(n-1)) + (-1)^{F(n+1)}f(n-3). \end{aligned}$$

[Continued on page 139.]