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SUMS OF PRODUCTS INVOLVING FIBONACCI SEQUENCES

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DEDICATED TO JANE LEGRANGE

Definition. $\{H_n\}$ is Fibonacci if $H_n = H_{n-1} + H_{n-2},\ n > 1.$ Every Fibonacci sequence $\{H_n\}$ can be written as $H_n = A\alpha^n + B\beta^n$, where α, β are the roots of $x^2 - x - 1 = 0.$ Thus

Theorem.

$$\sum_{i,j=0}^{n} a_{ij} H_i K_j = 0$$

for any two Fibonacci sequences if and only if

$$P(z,w) = \sum_{i,j=0}^{n} a_{ij} z^i w^j$$

vanishes on $\{(\alpha, \alpha), (\alpha, \beta), (\beta, \alpha), (\beta, \beta)\}$.

Example. (Berzsenyi [1]): If n is even, prove that

$$\sum_{k=0}^{n} H_k K_{k+2m+1} = H_{m+n+1} K_{m+n+1} - H_{m+1} K_{m+1} + H_0 K_{2m+1}\,.$$

The corresponding $P(z,w)$ is easily seen to satisfy the hypothesis of the theorem (using $\alpha\beta = -1$, $\alpha^2 - \alpha - 1 = 0$).

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