$$g_{n,n-1} = 1 - g_{n-2,n-1} - g_{n-1,n-1} = 1 - (f_{n-1} - 1)a - (1 - (f_{n+2} - 1)a) = f_n a,$$

and

$$g_{n,n} = 1 - g_{n,n-1} = 1 - f_n a.$$

Thus our claim is verified.

Now by considering the n^{th} horizontal plane sum of E, we see that a is uniquely determined. Hence E is unique, and thus $E = A_n$. This completes the proof of the theorem.

Constructions for other extremal matrices and additional properties of planar stochastic matrices can be found in [1, 2].

REFERENCES

1. R. A. Brualdi and J. Csima, "Stochastic Patterns," to appear.
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BELL'S IMPERFECT PERFECT NUMBERS

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A perfect number is one which, like 6 or 28, is the sum of its aliquot parts. Euclid proved that $2^{p-1}(2^p - 1)$ is perfect when $(2^p - 1)$ is a prime; and it has been shown that this formula includes all perfect numbers which are even.[1]

In Eric Temple Bell's fascinating book[2], the seven perfect numbers after 6 are listed as follows:

28, 496, 8128, 130816, 2096128, 33550336, 8589869056.

Checking these numbers by Euclid's formula, I found that

$$2^8(2^9 - 1) = 256 \times 511 = 130816$$

and

$$2^{10}(2^{11} - 1) = 1024 \times 2047 = 2096128.$$

However, $511 = 7 \times 73$; and $2047 = 23 \times 89$.

Inasmuch as 511 and 2047 are not primes, it follows that 130816 and 2096128 are not perfect numbers, and they should not have been included in Bell's list.

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[1] *Encyclopedia Britannica,* Eleventh Edition, Vol. 19, page 863.
[2] *The Last Problem,* Simon and Schuster, New York, 1961, page 12.

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