FORMULA DEVELOPMENT THROUGH FINITE DIFFERENCES

EXAMPLE.

1 + 2 + 4 + 5 + 7 + 8 + 27 + 53 + 104 + 204 = 415.

By formula

CONCLUSION

Finite differences have wide application in formula development. There are, of course, many situations in which the use of this method leads to difficulties which other procedures can obviate. But where applicable the results are often obtained with such facility that other procedures seem laborious by comparison.

A GOLDEN DOUBLE CROSTIC

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Use the definitions in the clue story which follows to write the words to which they refer; then enter the appropriate letters in the diagram to complete a quotation from a mathematician whose name appears in the last line of the diagram. The name of the book in which this quotation appeared and the author's last name appear as the first letters of the clue words. The end of each word is indicated by a shaded square following it.

CLUE STORY

The mystic Golden Section Ratio, $(1 + \sqrt{5})/2$, called ____(A-1, A-2)____(the latter most commonly), occurs in several propositions in ___(A-3, A-4)_____on line segments and ___(A-5)____. This Golden Cut fascinated the ancient Greeks, particularly the ___(D-1)___, who found this value in the ratio of lengths of segments in the __(D-2)____and ___(D-3)____ and who also made studies in ____(D-4)____. The Greeks found the proportions of the Golden Rectangle most pleasing to the eye as evidenced by the ubiquitous occurrence of this form in art and architecture, such as _____(C-1)____ or in sculpture as in the proportion occurs frequently in the forms of living things and is closely related to the growth patterns of plants, as _____(C-4, C-5, C-6)_____, in which occur ratios of Fibonacci numbers. The Golden Section is the limiting value of the ratio of two successive Fibonacci numbers (named for ____(G-1)____), being closely approximated by the ______.

By some mathematicians, the beauty of the <u>(N)</u> relating to the Golden Section is compared to the theorem of the <u>(D-1)</u> and to such results from projective geometry as those seen in Pascal's "Mystic <u>(B)</u>" or even in the applications of mathematics in the *Principia Mathematica* of <u>(I)</u> while the constant $(1 + \sqrt{5})/2$ itself is rivalled by <u>(E-1)</u> and <u>(E-2)</u>.

Unfortunately, not all persons find mathematics beautiful. <u>(H-1)</u> was one of the four branches of arithmetic given by the Mock Turtle in *Alice in Wonderland*, and the card player's description of the sequence 2, 1, 3, 4, 7, ______,18, 29, 47, ... would be <u>(H-2)</u>, while some have to have all mathematics of practical use, such as in reading an <u>(M)</u>.

[The solution appears on page 83 of the Quarterly.]

A GOLDEN DOUBLE CROSTIC

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A GOLDEN DOUBLE CROSTIC

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