(9) $$\overline{d(a)} = \lim_{n\to\infty} \frac{|I_1|n + o(n)}{\frac{n + \log(a+1)}{\log c} + o(n)} = \log(1 + 1/a)$$

and the desired conclusion follows.

REFERENCES

1. R. L. Duncan, "Note on the Initial Digit Problem," *The Fibonacci Quarterly,* Vol. 7, No. 5, pp. 474–475.
2. G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers,* 4th ed., 1960, p. 46.
3. J. G. Van der Corput, "Diophantische Ungleichungen I: Zur Gleich Verteilung Modulo Eins," *Acta Math.,* 1930-31 (378), pp. 55–56.
4. G. H. Hardy and E. M. Wright, *An Introduction to the Theory of Numbers,* 4th ed., 1960, p. 390.
5. H. Halberstam and K. F. Roth, *Sequences,* Oxford, 1966, Vol. I.

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ADDENDA TO ADVANCED PROBLEMS AND SOLUTIONS

These problem solutions were inadvertently skipped over for a few years. Our apologies.

FORM TO THE RIGHT

H-211 Proposed by S. Krishman, Orissa, India. (corrected)

A. Show that $\binom{2n}{n}$ is of the form $2n^3k + 2$ when n is prime and $n > 3$.

B. Show that $\binom{2n-2}{n-1}$ is of the form $n^3k - 2n^2 - n$, when n is prime.

$\binom{m}{j}$ represents the binomial coefficient, $\dfrac{m!}{j!(m-j)!}$.

Solution by P. Tracy, Liverpool, New York.

A. The Vandermonde convolution identity is $\binom{n}{m} = \Sigma\binom{n-L}{k}\binom{L}{m-k}$. Appling this to $\binom{2p}{p}$ (using $L = p$), we get

$$\binom{2p}{p} = \sum_{k=0}^{p}\binom{p}{k}^2 = 2 + \sum_{k=1}^{p-1}\binom{p}{k}^2.$$

Since p is a prime, $p \mid \binom{p}{k}$ for $k = 1, 2, \cdots, p-1$. Now

$$\binom{p}{k}^2 \equiv p^2 \left(\frac{(p-1)(p-2)\cdots(p-k+1)}{k!}\right)^2 \pmod{p^3}.$$

Also $(p-i)/i \equiv -1 \pmod p$ and so

$$\frac{1}{p^2}\sum_{k=1}^{p-1}\binom{p}{k}^2 \equiv \sum_{k=1}^{p-1}\frac{1}{k^2} \equiv 2 \qquad \text{quad. res. } (\text{mod } p)$$

(since every quadratic residue mod p has exactly two roots, $\pm a$). Let g be a primitive root, mod p, then the quadratic residues are

$$1, g^2, g^4, \cdots, g^{\frac{p-3}{2}}$$

To find the sum of the quadratic residues, we use the geometric sum formula to obtain $(g^{p-1} - 1)/(g^2 - 1)$. Note that $p > 3$ implies $g^2 - 1 \not\equiv 0 \pmod p$. Hence Σ quad. res. $\equiv 0 \pmod p$. Therefore

$$2p^3 \,\Big|\, \sum_{k=1}^{p-1}\binom{p}{k}^2 \quad \text{and} \quad \binom{2p}{p} \equiv 2 \pmod{2p^3}.$$

[Continued on page 165.]