This may be easily derived from (1) with $j = 1$ by applying (25). The critical step is

$$\nabla^{2k} L_k = L_{k-4k} = L_{-3k}$$

according to (25). We obtain

(35a) $$\nabla^{2k-1} G_k = L_{-3k+2}, \quad k \geq 1$$

(35b) $$\nabla^{2k} G_k = L_{-3k} + F_{-1}, \quad k \geq 1$$

(35c) $$\nabla^{2k+1} G_k = L_{-3k-2} + F_{-2}, \quad k \geq 0,$$

where, of course, $F_{-2} = -1$ and $F_{-1} = 1$. Equations (35) prove what is obvious by looking at Table 2, namely if we make a zig-zag below the 4 entry we obtain the sequence: $-1, 2, -3, 7, -12, 19, -29, 46, -75, 123, \cdots$ which is almost the Lucas sequence. This makes the whole sequence easy to generate by hand. Finally the choice of letter for these sequences was Gould's [1] who suggested my name for them after seeing my paper [6].

The author appreciates some comments by Zeitlin [8] concerning (14) and (23). Zeitlin [7] has also pointed out that the subscript of the subscript of the last term of Eq. (12) of [6] should be $(k-1)$ and not $(k-2)$. This misprint is obvious from the expansion in (13) of [6].

Having found that the messy looking $G_{j,k}$ sequence actually satisfies the near Fibonacci relationships (10) and (12) and further that the Lucas numbers have made their presence known, I am impelled to write down an old haiku of mine in which even the numbers of syllables in each line, namely 3, 2, 5, 7 are themselves a Fibonacci sequence.

PHI

Multiply
Or add
We always reach phi
Symmetries we perpetrate.

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[Continued from page 165.]

where the i^{th} column of C_n is the i^{th} row of Pascal's triangle adjusted to the main diagonal and the other entries are 0's. Find $C_n \cdot A_n^T$.

Solution by P. Bruckman, University of Illinois at Chicago, Chicago, Illinois.

A. Let $B_n = A_n \cdot A_n^T$. Let a_{ij} and b_{ij} be the entries in the i^{th} row and j^{th} column of A_n and B_n, respectively. Similarly, let a_{ij}^T be the j^{th} entry of A_n^T. Then

$$a_{ij} = \binom{i-1}{j-1} \quad \text{if } i \geq j;$$
$$= 0 \qquad \text{elsewhere;}$$

therefore,

$$a_{ij}^T = \binom{j-1}{i-1} \quad \text{if } i \leq j$$
$$= 0 \qquad \text{elsewhere.}$$

[continued on page 183.]