at the grid, the maximum stable size, and the bed depth over which the bubbles may grow from their initial to their stable diameter. Once having reached their maximum stable diameter any further unlikely mergers would also lead to collapse, so that bubble diameter may be considered constant once having reached the stable size.

An unquestionably conservative approach to a minimal risk pilot plant reactor free of scaleup considerations would suggest it equal the larger of either "cloud" or "shell" diameter surrounding the system's maximum stable bubble.

NOMENCLATURE

C_D = Drag coefficient, dimensionless
D_B = Bubble diameter, feet
D_{Bi} = Bubble diameter at grid level
g = Gravitational acceleration, 32.2 ft./sec.^{2}
L_B = Bed depth, feet
P = Grid jet penetration
Re = Reynolds number, dimensionless
V_B = Bubble rise velocity, ft./sec.
V_{mf} = Incipient fluidization velocity, ft./sec.
ρ_B = Bed density, lbs./cu. ft.
ρ_G = Gas density, lbs./cu. ft.

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Thus,

$$b_{ij} = \sum_{k=1}^{n} a_{ik}a_{kj}^T = \sum_{k=1}^{n} \binom{i-1}{k-1}\binom{j-1}{j-k}.$$

Actually, the effective upper limit of this last summation is min. $(i,j) = m + 1$. Therefore,

$$b_{ij} = \sum_{k=0}^{m} \binom{i-1}{k}\binom{j-1}{j-1-k} = \sum_{k=0}^{m} \binom{j-1}{k}\binom{i-1}{i-1-k},$$

which shows that b_{ij} is symmetric in i and j.

Actually, the last summation may readily be evaluated by the Vandermonde convolution theorem, so that:

(1) $$b_{ij} = \binom{i+j-2}{i-1}, \quad \text{for all } i,j \leq n.$$

B. As before, let $D_n = C_n \cdot A_n^T$; let c_{ij} and d_{ij} be the entries in the i^{th} row and j^{th} column of C_n and D_n, respectively. Then

$$c_{ij} = \binom{j-1}{i-j}, \qquad j \leq i \leq 2j-1$$

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