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[Continued from page 146.]

and that $V_n = 2$ satisfies

$$V_{n+2} = 2V_{n+1} - V_n,$$

we can rewrite (1.2) as

$$2\sum_{k=1}^{n} U_k^3 = V_n\left(\sum_{k=1}^{n} U_k\right)^2.$$

This suggests the following result for integer sequences.

Conjecture. Let U_k, with $U_0 = 0$, $U_1 = 1$, and V_k, with $V_0 = 2$, $V_1 = P$, be two solutions of

$$W_{k+2} = PW_{k+1} + QW_k, \qquad k = 0, 1, \cdots,$$

where P and Q are integers with $P \geqslant 2$ and $P + Q \geqslant 1$. We then claim that

$$2\sum_{k=1}^{n} U_k^3 \leqslant V_n\left(\sum_{k=1}^{n} U_k\right)^2 \qquad (n = 1, 2, \cdots). \tag{3.1}$$

Remarks. For $P = 2$ and $Q = -1$, (3.1) gives (1.2). Using double induction, one can prove the conjecture for $P + Q \geqslant 3$, which leaves the two cases $P + Q = 2$ and $P + Q = 1$ open.

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