FIBONACCI, INSECTS, AND FLOWERS

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It remains an interesting phenomenon that elements of the Fibonacci and Lucas sequences appear in numerous structural entities belonging to varied species of higher plants. McNabb [3] cites the abundance of flower species with numbers of petals (up to 89 in Michaelmas daisies) that correspond to Fibonacci numbers. Karchmar [1] obtained the commonly observed angle between adjacent leaf primordia (137°30') by applying the limiting value of the following Fibonacci ratio:

$$\frac{F_n}{F_{n+1}} \tag{1}$$

where F_n and F_{n+1} denote, respectively, the nth and $(n+1)$th elements of the Fibonacci sequence.

Although there exists a considerable body of literature pertaining to plant structure and Fibonacci sequences, the above references are singled out for their use of expression (1). As pointed out by McNabb [3], phyllotaxic descriptions are often denoted in the form of expression (1). It is to expression (1) that we give most of our concern in relation to insects which reside on flowers of field thistle (*Circium discolor*). Specifically, we are interested in the sequences of lengths among these insects. Table 1 lists the species of insect, sample size, mean length, and standard deviation.

Table 1

Length Statistics of Five Insect Species Resident on
Flowers of *Circium discolor*

Insect	Sample Size	Mean Length (mm)	Standard Deviation
Diabrotica longicornis (beetle)	15	6.0	0.58
Plagiognathus (bug)	13	3.7	0.23
Olibrus semistriatus (beetle)	17	2.2	0.25
Orius insidiosus (bug)	14	2.0	0.10
Frankliniella tritici (thrip)	15	0.9	0.12

Let us assume that because flowers are of a limited volume, insects are competing for space. Another alternative is that of competition for food, but since we rarely observe flowers devastated by insects, we presently reject this alternative. We can further speculate that if competition is for space, we expect the appearance of ecological and evolutionary mechanisms

aimed at the avoidance of physical encounter. Such an avoidance may be re-
alized if each insect were to possess a "refuge" (i.e., a volumetric space)
for the avoidance of larger insects. Within a complex flower, such as field
thistle, smaller insects could avoid larger insects by seeking crevices which
larger insects could not enter. This mechanism does not exclude other means
of avoidance, although if we accept the mechanism of avoidance by spatial
refuge, then there should arise constraints on the size of each insect spe-
cies. We can thus imagine that, of a pair of insects, the larger will "push"
the smaller (over evolutionary time) to a reduced size. We assume here that,
upon encounter, the smaller insect is more likely to move away from the lar-
ger than the larger move away from the smaller. In this manner, the largest
insect residing on the flower will determine, at a first approximation, the
entire size sequence of the remaining insects.

From the above consideration, we make use of the Fibonacci sequence in an
unusual manner. Since it is assumed that the largest insect determines the
length sequence, we start our sequence backwards, setting our largest number
as the first term in the sequence. We then define our sequence, on the basis
of the first term (u_1), as:

$$u_n = u_1(\beta^{n-1}) \tag{2}$$

where $\beta = 1/\alpha$ and α (the Fibonacci ratio in the limit) approximates the value
of 1.62. Thus, $\beta \doteq .62$.

We are now able, given the first term, to calculate elements of (2). Re-
calling that the length of the largest insect is 6.0 mm, we may set this
value as the first term in the sequence, and then proceed to calculate the
next four terms. A comparison of the empirical and predicted sequences is
impressive.

Predicted Sequence	Empirical Sequence
6.0	6.0
3.7	3.7
2.3	2.2
1.4	2.0
0.9	0.9

We may imply from this comparison that the length ratio of two neighbor-
ing insects in the sequence, taking the larger to the smaller, should approx-
imate 1.62. This ratio can then be viewed as a "limiting similarity" [2] for
two species, i.e., how similar can two species be in the utilization of a re-
source (this resource being space in our consideration) before one excludes
the other.

If we accept the above comparison of sequences as noncoincidental, we can
go on to hypothesize that the refuge volumes occupied by these five species
of insects may be a function of the insects' lengths. If the volume occupied
is simply related to the insect's length by a constant (k), then we can de-
note a volume sequence (u_n') as:

$$u_n' = ku_1(\beta^{n-1})$$

which is qualitatively identical to (2). That is, these insects may possess refuge volumes which correspond, in magnitude, to a Fibonacci sequence.

ACKNOWLEDGMENTS

REFERENCES

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3. Sr. M. D. McNabb, "Phyllotaxis," *The Fibonacci Quarterly*, Vol. 1, No. 4 (December 1963), pp. 57-60.

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