1979] ADDENDA TO "PYTHAGOREAN TRIPLES CONTAINING FIBONACCI NUMBERS:" 293

REFERENCES


###

ADDENDA TO "PYTHAGOREAN TRIPLES CONTAINING FIBONACCI NUMBERS: SOLUTIONS FOR $F_n^2 + F_k^2 = K^2"

MARJORIE BICKNELL-JOHNSON
A. C. Wilcox High School, Santa Clara, CA 95051

In a recent correspondence from J. H. E. Cohn, it was learned that Ljunggren [1] has proved that the only square Pell numbers are 0, 1, and 169. (This appears as an unsolved problem, H-146, in [2] and as Conjecture 2.3 in [3].) Also, if the Fibonacci polynomials $B_n(x)$ are defined by

$$B_0(x) = 0, B_1(x) = 1, \text{ and } B_{n+2}(x) = xB_{n+1}(x) + B_n(x),$$

then the Fibonacci numbers are given by $B_n = B_n(1)$, and the Pell numbers are $P_n = B_n(2)$. Cohn [4] has proved that the only perfect squares among the sequences $B_n(a)$, $a$ odd, are 0 and 1, and whenever $a = k^2$, $a$ itself. Certain cases are known for $a$ even [5].

The cited results of Cohn and Ljunggren mean that Conjectures 2.3, 3.2, and 4.2 of [3] are true, and that the earlier results can be strengthened as follows.

If $(n,k) = 1$, there are no solutions in positive integers for

$$P_n^2(a) + P_k^2(a) = K^2, \quad n > k > 0,$$

when $a$ is odd and $a \geq 3$.

This is the same as stating that no two members of $B_n(a)$ can occur as the lengths of legs in a primitive Pythagorean triangle, for $a$ odd and $a \geq 3$.

When $a = 1$, for Fibonacci numbers, if

$$P_n^2 + P_k^2 = K^2, \quad n > k > 0,$$

then $(n,k) = 2$, and it is conjectured that there is no solution in positive integers. When $a = 2$, for Pell numbers, $P_n^2 + P_k^2 = K^2$ has the unique solution $n = 4, k = 3$, giving the primitive Pythagorean triple 5-12-13.

REFERENCES


###