(3°) The Blissard generating function y_r of column r is given by use of (2):

$$\sum_m \binom{m}{r} u_{m-r}\frac{x^m}{m!} = \frac{x^r}{r!}\sum_m u_{m-r}\frac{x^{m-r}}{(m-r)!},$$

so that

$$y_r = e^{-x}(1-x)^{-2}x^r/r!.$$

(4°) The sum $\sum_{r=0}^{+\infty} y_r$ is $(1-x)^{-2} = 1 + 2x + 3x^2 + \cdots$, which confirms that the coefficient of $x^m/m!$ is $(m+1)!$.

(5°) According to (3), the ratio $u_m/(m+1)!$ is equal to

$$1 - \binom{m}{1}\frac{1}{m+1} + \binom{m}{2}\frac{1}{(m+1)m} - \cdots + (-1)^r\binom{m}{r}\frac{1}{(m+1)_r} + \cdots$$

As m increases, with fixed r, the general term of this sum tends toward $(-1)^r/r!$; it follows that the sum itself tends toward e^{-1}, which is the limiting proportion of irregular permutations.

(6°) Using (2), it appears that

$$\frac{u(m, r)}{(m+1)!} = \frac{u_{m-r}}{(m-r+1)!}\cdot\frac{m-r+1}{r!(m+1)}.$$

As m increases, the second member tends toward $e^{-1}/r!$. The latter result means that, if a permutation is chosen at random in S_{m+1} and if m increases, the limiting probability distribution of its regularity is a Poisson distribution with mean 1.

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STAR POLYGONS, PASCAL'S TRIANGLE, AND FIBONACCI NUMBERS

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In recent years, there has been some flurry of excitement over the relationship between the complexity of a graph, i.e., the number of distinct spanning trees in a graph, and the Fibonacci and Lucas numbers [1, 2]. In this note, I shall demonstrate a relationship, although incomplete, between the Fibonacci numbers and the star polygons. My hope is to spur further research into the connection between nonplanar graphs and their enumeration from number theory.

The star n-polygon $\left\{ {n \atop d} \right\}$, one of the simplest of these nonplanar graphs, is constructed by placing n points equidistantly on the perimeter of a circle and then connecting every dth point such that

$$n/d \text{ is relatively prime and } n \neq n - d \neq 1.$$

The last condition effectively removes the class of all regular polygons.

The group structure of such polygons is clear; it is related to the partition of unity in which this partition is prime. Therefore, it does not come as any surprise that a symmetry relation for the star n-polygon $\left\{ {n \atop d} \right\}$ is

$$\left\{ {n \atop d} \right\} = \left\{ {n \atop n - d} \right\}. \tag{1}$$

This fact was brought to my attention by Ms Dianne Olvera.

It then intrigued me to discern whether the symbolic symmetry exhibited by (1) could be generated by a somewhat similar number-theoretic symmetry, that produced by Pascal's triangle; row-wise, the combinatorial symmetry

$$\binom{\gamma}{\beta} = \binom{\gamma}{\gamma - \beta} \tag{2}$$

exists.

At first glance, the similarity between (1) and (2) appears to be only cosmetic, since there are absolutely no restrictions on the values of the positive integers γ and β as long as $\gamma > \beta$. Secondly, there seems to be no numerical congruence between (1) and (2).

On the other hand, if one were to examine the Fibonacci numbers F_n generated by summing entries along the diagonals of Pascal's triangle, an algorithm can be constructed that will produce all the possible star n-polygons excluding a sparse set. The procedure is as follows.

Algorithm: The symmetry relation $\left\{ {n \atop d} \right\} = \left\{ {n \atop n - d} \right\}$ for star n-polygons results from partitioning any number or sum of numbers in the sum of some Fibonacci sequence equalling n around its relatively prime divisors.

Example 1: The star pentagons (pentagrams) $\left\{ {5 \atop 2} \right\} = \left\{ {5 \atop 3} \right\}$ are generated by summing the Fibonacci numbers $F_3 + F_4 = 5$. Since its prime divisors are 2 and 3, respectively, partitioning 5 around 2 yields the star pentagon $\left\{ {5 \atop 2} \right\} = \left\{ {5 \atop 3} \right\}$.

Example 2: The star heptagons $\left\{ {7 \atop 5} \right\} = \left\{ {7 \atop 2} \right\}$ and $\left\{ {7 \atop 3} \right\} = \left\{ {7 \atop 4} \right\}$ are generated by summing the Fibonacci numbers $F_1 + F_2 + F_3 + F_4 = 1 + 1 + 2 + 3 = 7$. Partitioning the sum around 3 produces $\left\{ {7 \atop 3} \right\} = \left\{ {7 \atop 4} \right\}$. The reader can quickly convince himself or herself that partitions around various alternative sums of this sequence which are relatively prime to 7 do not generate any other possibilities.

Example 3: Star nonagons are obtainable by summing the sequences

$$F_1 + F_4 + F_5 = 1 + 3 + 5 = 9$$

and

$$F_1 + F_2 + F_3 + F_5 = 1 + 1 + 2 + 5 = 9.$$

The former yields, upon partitioning around the sum $F_1 + F_4$, the star nonagons $\left\{ {9 \atop 4} \right\} = \left\{ {9 \atop 5} \right\}$, while the latter yields, upon partitioning around the sum $F_1 + F_2 + F_3$ or around F_3, the previous star nonagon or $\left\{ {9 \atop 2} \right\} = \left\{ {9 \atop 7} \right\}$.

I have examined all the possible star nonagons for all n inclusive of 21. When $n = 13$ and 21, this algorithm breaks down and will not produce $\left\{ {13 \atop 6} \right\}$, $\left\{ {21 \atop 4} \right\}$, and $\left\{ {21 \atop 10} \right\}$. For larger values of n, other discrepancies will appear (n need not be a Fibonacci number), but always much fewer in number than the star n-gons that are generated.

It therefore appears that the Fibonacci sequence on its own cannot exhaustively generate all star n-gons. The basic reason for this nonisomorphism is that the Fibonacci numbers are related to the combinatorics of spanning trees, the combinatorics of planar graphs, not of nonplanar graphs.

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A CONVERGENCE PROOF ABOUT AN INTEGRAL SEQUENCE

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ABSTRACT

The major theorem proven in this paper is that every positive integer necessarily converges to 1 by a finite number of iterations of the process such that, if an odd number is given, multiply by 3 and add 1; if an even number if given, divide by 2.

The first step is to show an infinite sequence generated by that iterative process is recursive. For the sake of that object, an integral variable x with $(\ell + 1)$ bits is decomposed into $(\ell + 1)$ variables $a_0, a_1, \ldots, a_\ell$, each of which is a binary variable. Then, rth iteration, starting from x, has a correspondence with a fixed polynomial of $a_0, \ldots, a_\ell$, say

$$f_r(a_0, \ldots, a_\ell),$$

no matter what value x takes. Since the number of distinct f_r's is finite in the sense of normalization, the common f_r must appear after some iterations. In the circumstances, the sequence must be recursive.

The second step is to show that a recursive segment in that sequence is (1, 2) or (2, 1). For that object, the subsequences with length 3 of that segment are classified into twelve types concerned with the middle elements modulo 12. The connectability in the segment with length 5 or larger, and the constancy of the values at the head of each segment, specify the types of subsequences, found impossible, as well as with lengths 1, 3, and 4. The only possible segment is that with length 2, like (1, 2) or (2, 1).