$+ v(n)F_{n+3} + A_p$, where u and v are polynomials in n of degree p and A_p is a constant independent of n. It can be shown that the coefficients of u and v may be found by solving the $2p + 2$ equations obtained by letting n take on any $2p + 2$ consecutive values.

Also solved by Zvi Dresner and Marjorie Bicknell

A CLASSICAL SOLUTION

H-16 *Proposed by H. W. Gould, West Virginia University, Morgantown, W. Va.*

Define the ordinary Hermite polynomials by $H_n = (-1)^n e^{x^2} D^n (e^{-x^2})$.

Show that:

(i) $$\sum_{n=0}^{\infty} H_n(x/2)\frac{x^n}{n!} = 1 ,$$

(ii) $$\sum_{n=0}^{\infty} H_n(x/2)\frac{x^n}{n!}F_n = 0 ,$$

(iii) $$\sum_{n=0}^{\infty} H_n(x/2)\frac{x^n}{n!}L_n = 2e^{-x^2} ,$$

where F_n and L_n are the n^{th} Fibonacci and n^{th} Lucas numbers, respectively.

We recall that $\sum_{n=0}^{\infty} H_n(t)\frac{x^n}{n!} = e^{2tx-x^2}$. For $t = \frac{x}{2}$ this reduces to $\sum_{n=0}^{\infty} H_n\left(\frac{x}{2}\right)\frac{x^n}{n!} = 1$.

Put $\alpha = \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1-\sqrt{5}}{2}$. Then $(\alpha - \beta)\sum_{n=0}^{\infty} H_n\left(\frac{x}{2}\right)\frac{x^n}{n!}F_n = e^{(\alpha-\alpha^2)x^2} - e^{(\beta-\beta^2)x^2} = 0$ since $\alpha - \alpha^2 = \beta - \beta^2 = -1$.

Similarly,

$$\sum_{n=0}^{\infty} H_n\left(\frac{x}{2}\right)\frac{x^n}{n!}L_n = e^{(\alpha-\alpha^2)x^2} + e^{(\beta-\beta^2)x^2} = 2e^{-x^2} .$$

See also the solution in the last issue by Zvi Dresner.

Reference continued from page 44.

1. K. F. Roth, "Rational Approximations to Algebraic Numbers," Mathematika 2 (1955) pp. 1 – 20, p. 168.