LETTER TO THE EDITOR

The Editor, Fibonacci Quarterly.

Dear Dr. Hoggatt,

I refer to the article, "Dying Rabbit Problem Revived" in the December 1963 issue. The solution given there is patently wrong — if only because the alleged number of rabbits tends to minus infinity as n tend to infinity. It may easily be shown that the correct answer, $X_n$, is given by the recurrence relation

$$X_{n+3} = X_{n+2} + X_{n+1} - X_n, \quad n \geq 0$$

together with the initial conditions

$$X_n = F_{n+1} \quad \text{for } n = 0, 1, \ldots, 11; \quad X_{12} = 232.$$

In view of the fact that the two equations $z^2 - z - 1 = 0$ and $z^{13} - z^{12} - z^{11} + 1 = 0$ have no common root, it is clear that the answer can never be expressed simply as a linear expression in Fibonacci and Lucas numbers whose coefficients are merely polynomials in n. For, any such expression, $Y$, where the highest power of n which occurs is $n^m$, satisfies

$$(E^2 - E - 1)^{m+1} Y = 0.$$

In particular the expression found by Bro. Alfred satisfies

$$(E^2 - E - 1)^2 Y = 0.$$

The error made by Bro. Alfred stems from his table on p. 54 where the number of dying rabbits in the $(n+3)$th month is seen to be $F_n$ for $n = 1, 2, \ldots, 11$ and it is then assumed without proof that this is true for other values of n. In fact the very next but one value on n, namely $n = 13$ shows that this is false. In fact of course the number of dying rabbits in the $(n+3)$th month equals the number of bred rabbits in the $(n+1)$th month, and this will be less than $F_n$ for all n exceeding 12.

Yours sincerely, (John H. E. Cohn)

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