FIBONACCI GEOMETRY

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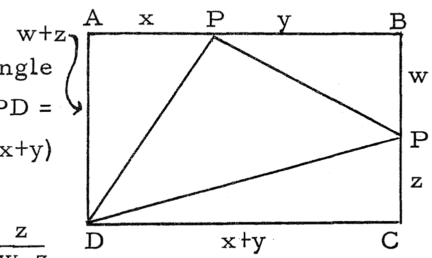
Below are some additional observations about Hunter's [1] article.



If the rectangle ABCD has a triangle DPP' inscribed within it so that $\triangle APD = \triangle BPP' = \triangle P'DC$ then $x(w+z) = wy = z(x+y)$ whence

(i) $$\frac{y}{x} = \frac{w+z}{w} = \frac{z}{w-z}$$

(ii) $$\therefore w^2 - z^2 = wz, \text{ i.e., } w^2 - wz - z^2 = 0 \quad w = \frac{z \pm \sqrt{z^2+4z^2}}{2} = \varphi z$$

(iii) From (i) $$\frac{y}{x} = \varphi \text{ or } y = \varphi x$$

Thus P, P' divide their sides in the Golden Section.

Now, suppose ABCD is the Golden Rectangle, beloved of the Greek architects, i.e. $AB/BC = \varphi$, then $\frac{x+y}{w+z} = \varphi$. Hence, from (ii) and (iii) $\frac{x(1+\varphi)}{z(1+\varphi)} = \varphi$, i.e. $x = \varphi z$ whence $x = w$. From (i) $y = w+z = \varphi^2 z$. Since $\sphericalangle A = \sphericalangle B = rt\sphericalangle$ and $x = w$, $y = w+z$, triangles PAD, P'BP are congruent. It follows that PD = PP', that $\sphericalangle APD$ is the complement of $\sphericalangle BPP'$, whence $\sphericalangle DPP'$ is a right angle.

The area of the right triangle is

$$\frac{1}{2}(w^2+y^2) = \frac{1}{2}(\varphi^2 z^2 + \varphi^4 z^2) = \frac{1}{2}\varphi^2 z^2(\varphi^2+1) = \frac{1}{2}z^2(\varphi+1)(\varphi+2) = \frac{1}{2}z^2(4\varphi+3).$$

we may conclude, therefore, that if the rectangle is the Golden Rectangle, that is, if its adjacent sides are in the Golden Ratio, φ, then the inscribed triangle is right-angled and isosceles, the length of the equal sides being $z\sqrt{4\varphi+3}$.

Editorial Note: PP' || AC

REFERENCES

1. J. A. H. Hunter, "Triangle Inscribed in a Rectangle" 1(1963) October, pg. 66.