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$k = F_{n+1}$, we have that $k+1 = F_{n+1} + F_2$, and we are through. If $k = F_{n+1} - 1$, we have $k+1 = F_{n+1}$ and we are through. If $k = F_{n+1} - 2$ we have $k+1 = F_{n+1} - 1$, which by Lemma 1 (A or B), can be represented as claimed and we are through again. Therefore let us consider $k \leq F_{n+1} - 3$.

Now the representation for k in this form can best be expressed as $k = F_n + a_{n-2} F_{n-2} + a_{n-3} F_{n-3} + a_{n-4} F_{n-4} + \cdots + a_3F_3 + a_2F_2$ where $a_i = 0$ or 1 for $2 \leq i \leq n-2$, and $a_i = 1$, implies that $a_i \pm 1 = 0$. Now there are only two possibilities for a_2 and a_3 in this representation. Either $a_2 = a_3 = 0$, or $a_2 \neq a_3$. If the first case is true for k, we can represent k+1 in the required manner, simply by adding 1 to k in the form of $a_2 = 1$. If the second case is true for k, we then claim that there exists at least one place in the representation where $a_i = a_{i+1} = 0$, since otherwise, $k = F_{n+1} - 1$ which we have already taken care of above.

Therefore we can represent k+1 by the following:

$$k+1 = F_n + a_{n-2}F_{n-2} + \cdots + a_{i+2}F_{i+2} + a_{i-1}F_{i-1} + \cdots + a_3F_3 + a_2F_2 + 1$$

Now consider the expression from $a_{i+2}F_{i+2}$ on and the resulting inequality.

$$a_{i+2}F_{i+2} + a_{i-1}F_{i-1} + \cdots + a_3F_3 + a_2F_2 + 1 \leq F_{i+3} - 1 \leq F_{n-1} - 1,$$

by our Inductive Assumption. Also by the Inductive Assumption, we can represent the expression from $a_{i+2}F_{i+2}$ on in the proper form which implies that we can then also represent k+1 in the proper form. This shows that the proof holds for all positive integers N. Q.E.D.

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