where B(s, t) is given by (3.7), and ϕ and ψ are the differential operators

$$\phi = s\frac{\partial}{\partial s}, \quad \psi = t\frac{\partial}{\partial t} . \tag{5.3}$$

I wish to thank Prof. Paul F. Byrd for his many helpful suggestions during the preparation of this article and the two previous ones.

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ASSOCIATIVITY AND THE GOLDEN SECTION

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E. T. Bell, A functional equation in arithmetic, Trans. Amer. Math. Soc., 39(1936), 341-344, gave a discussion of some matters suggested by the functional equation of associativity

$$\varphi(x, \varphi(y, z)) = \varphi(\varphi(x, y), z) .$$

As a prelude, Bell noted the following theorem.

THEOREM 1. The only polynomial solutions of $\varphi(x, \varphi(y, z)) = \varphi(\varphi(x, y), z)$ in the domain of complex numbers are the unsymmetric solutions $\varphi(x, y) = x$, $\varphi(x, y) = y$, and the symmetric solution

$$\varphi(x, y) = a + b(x + y) + cxy ,$$

in which a, b, c, are any constants such that $b^2 - b - ac = 0$.

It is amusing to note a special case. The operation defined by

$$x * y = 1 + b(x + y) + xy$$

is associative only if $b = \frac{1}{2}(\pm\sqrt{5})$.

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