In a course in differential equations, solving
\[
\frac{d^2 y}{dx^2} - \frac{dy}{dx} - y = 0 \quad (y = 0; \; y' = 1, \; x = 0)
\]
leads to
(1) \[ y = \frac{e^{\alpha x} - e^{\beta x}}{\alpha - \beta} = \sum_{n=0}^{\infty} \frac{\alpha^n - \beta^n}{\alpha - \beta} \frac{x^n}{n!}, \]
where \( \alpha = (1 + \sqrt{5})/2 \) and \( \beta = (1 - \sqrt{5})/2 \)
both satisfy the auxiliary equation \( m^2 - m - 1 = 0 \).

On the other hand, solving this same problem directly in infinite series of the form
(2) \[ y = \sum_{n=0}^{\infty} a_n x^n \]
leads to the recurrence relation
\[
(n+2)(n+1)a_{n+2} - (n+1)a_{n+1} - a_n = 0,
\]
with \( a_0 = 0, \; a_1 = 1. \)

If we set \( a_n = \frac{u_n}{n!} \) this becomes
\[
(n+2)(n+1) \frac{u_{n+2}}{(n+2)!} - \frac{(n+1)u_{n+1}}{(n+1)!} - \frac{u_n}{n!} = 0
\]
or
\[
\frac{u_{n+2}}{(n+2)!} - \frac{u_{n+1}}{(n+1)!} - \frac{u_n}{n!} = 0,
\]
with \( u_0 = 0 \) and \( u_1 = 1. \)

Thus \( a_n = \frac{u_n}{n!} = \frac{F_n}{n!} \),
where \( F_n \) is the \( n \)th Fibonacci number.

Substituting these values of \( \frac{u_n}{n!} = a_n \) into (2) yields
(3) \[ y = \sum_{n=0}^{\infty} \frac{F_n}{n!} x^n \]

Continued on page 196.