The purpose of this note is to point out a connection between the Fibonacci sequence and rows of Pascal's triangle. It is known that

\[ F_n = \frac{1}{\sqrt{5}} \left[ \left( \frac{1 + \sqrt{5}}{2} \right)^n - \left( \frac{1 - \sqrt{5}}{2} \right)^n \right] \]

Expanding by the binomial theorem and collecting terms we get

\[ F_n = \frac{\binom{n}{1} + \binom{n}{3} 5 + \binom{n}{5} 5^2 + \binom{n}{7} 5^3 + \binom{n}{9} 5^4 + \cdots}{2^{n-1}} \]

But \( 2^{n-1} = \binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \binom{n}{6} + \binom{n}{8} + \cdots \)

Therefore

\[ F_n = \frac{\binom{n}{1} + \binom{n}{3} 5 + \binom{n}{5} 5^2 + \binom{n}{7} 5^3 + \binom{n}{9} 5^4 + \cdots}{\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \binom{n}{6} + \binom{n}{8} + \cdots} \]

As for the Lucas sequence it is known that

\[ L_n = \left( \frac{1 + \sqrt{5}}{2} \right)^n + \left( \frac{1 - \sqrt{5}}{2} \right)^n \]

Expanding as above and collecting terms and remembering that \( 2^{n-1} \) is also equal to \( \binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \binom{n}{7} + \binom{n}{9} + \cdots \) we get

\[ L_n = \frac{\binom{n}{1} + \binom{n}{3} 5 + \binom{n}{5} 5^2 + \binom{n}{7} 5^3 + \binom{n}{9} 5^4 + \cdots}{\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \binom{n}{7} + \binom{n}{9} + \cdots} \]

Note the exchange of binomial coefficients in the two formulas. Numerical examples: To find \( F_7 \) we look in row 7 of Pascal's Triangle and find

\[ \frac{7 + 35 \cdot 5 + 21 \cdot 5^2 + 1 \cdot 5^3}{1 + 21 + 35 + 7} = \frac{832}{64} = 13 \]

Similarly for the 7th Lucas number

\[ \frac{1 + 21 \cdot 5 + 35 \cdot 5^2 + 7 \cdot 5^3}{7 + 35 + 21 + 1} = \frac{1856}{64} = 29 \]