FIBONACCI AND PASCAL

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The purpose of this note is to point out a connection between the Fibonacci sequence and rows of Pascal's triangle. It is known that

$$F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

Expanding by the binomial theorem and collecting terms we get

$$F_n = \frac{\binom{n}{1} + \binom{n}{3}5 + \binom{n}{5}5^2 + \binom{n}{7}5^3 + \binom{n}{9}5^4 + \cdots}{2^{n-1}}$$

But $$2^{n-1} = \binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \binom{n}{6} + \binom{n}{8} + \cdots$$

Therefore

$$F_n = \frac{\binom{n}{1} + \binom{n}{3}5 + \binom{n}{5}5^2 + \binom{n}{7}5^3 + \binom{n}{9}5^4 + \cdots}{\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \binom{n}{6} + \binom{n}{8} + \cdots}$$

As for the Lucas sequence it is known that

$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$$

Expanding as above and collecting terms and remembering that 2^{n-1} is also equal to $\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \binom{n}{7} + \binom{n}{9} + \cdots$ we get

$$L_n = \frac{\binom{n}{0} + \binom{n}{2}5 + \binom{n}{4}5^2 + \binom{n}{6}5^3 + \binom{n}{8}5^4 + \cdots}{\binom{n}{1} + \binom{n}{3} + \binom{n}{5} + \binom{n}{7} + \binom{n}{9} + \cdots}$$

Note the exchange of binomial coefficients in the two formulas. Numerical examples: To find F_7 we look in row 7 of Pascal's Triangle and find

$$\frac{7 + 35 \cdot 5 + 21 \cdot 5^2 + 1 \cdot 5^3}{1 + 21 + 35 + 7} = \frac{832}{64} = 13$$

Similarly for the 7th Lucas number

$$\frac{1 + 21 \cdot 5 + 35 \cdot 5^2 + 7 \cdot 5^3}{7 + 35 + 21 + 1} = \frac{1856}{64} = 29$$

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