2. S. L. Basin and V. E. Hoggatt, "A Primer on the Fibonacci Sequence — Part II," Fibonacci Quarterly, 1(1963), No. 2, pp. 61-68.
3. Dov Jarden, Recurring Sequences, Riveon Lematematika, 1958, pp. 42-44.

Additional Reading

4. R. Bellman, Introduction to Matrix Analysis, McGraw-Hill, 1960, pp. 228-229 (Kronecker Powers of Matrices.)

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THE GOLDEN CUBOID

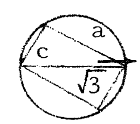
H. E. HUNTLEY

The problem of finding the dimensions of a cuboid (rectangular parallelopiped) of unit volume, having a diagonal 2 units in length leads to an interesting result.

Suppose the lengths of the edges are $\underline{a}$, $\underline{b}$ and $\underline{c}$. Then

$$(1) \quad a \cdot b \cdot c = 1 \quad \text{and} \quad (2) \sqrt{(a^2 + b^2 + c^2)} = 2$$

If only the ratios of these lengths are required, we may, without loss of generality, write $\underline{b} = 1$, provided that $\underline{a} \cdot \underline{c}$ can have the value unity and that $\underline{a}^2 + \underline{c}^2 = 3$. Now it is evident from Fig. 1, which represents the base of the cuboid, that the maximum value of $\underline{a} \cdot \underline{c}$ occurs when $a = c = \sqrt{3/2}$, so that $\underline{a} \cdot \underline{c}$ may have any value from zero to 3/2.



base

Fig. 1

Substituting $c = 1/a$ from (1) in (2), we have

$$a^2 + \frac{1}{a^2} = 3 \quad \text{i.e.,} \quad a^4 - 3a^2 + 1 = 0, \quad \text{whence}$$

$$a^2 = \frac{3 + \sqrt{5}}{2} = 1 + \varphi = \varphi^2,$$

so that $a = \varphi$, the Golden Section. The positive solution of the equation $x^2 - x - 1 = 0$ and the value of u_n/u_{n-1} as $n \to \infty$, where u_n is a member of the Fibonacci Series.

From (1) it follows that $c = \varphi^{-1}$, so that the required ratios are $a:b:c = \varphi:1:\varphi^{-1}$. It is easily verified that $\varphi^2 + 1 + \varphi^{-2} = 4$.

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