## ELEMENTARY PROBLEMS AND SOLUTIONS

Editorial note. Let  $R_n = F_{n-1}/F_n$ . It is well known and easily proved that  $R_2 > R_4 > R_6 > \dots R_7 > R_5 > R_3$ . This shows that the n for which  $R_n = \frac{1}{2}$  is unique.

Also solved by Charles R. Wall, Texas Christian University, Ft. Worth, Texas and the proposer.

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Moreover, these are the dimensions of the cuboid of unit volume, for  $\varphi \ge 1 \ge \varphi^{-1} = 1$ .



Certain other properties of the Golden Cuboid may be noted.

1. It is clear from Fig. 2 that the ratios of the areas of the faces are: AE:AC:CE =  $\varphi$ :1: $\varphi$ <sup>-1</sup>.

2. The total surface area of the cuboid is  $3(\varphi + 1 + \varphi^{-1}) = 6\varphi$ 

3. Four of the six faces of the cuboid are Gold Rectangles, e.g., CE (Fig. 3)

4. Each of the four diagonals of the cuboid is inclined to the base at an angle of  $30^{\circ}$ .

5. The ratio of the area of the sphere circumscribing the cuboid to that of the cuboid is  $2\pi:3\varphi$ .

One further point is of interest.

6. It is well known that, if a square CK is cut off from the Golden Rectangle CE (Fig. 3), the sides of the remaining rectangle LE are also in the ratio  $\varphi$ :1. And of course the dissection may be repeated until the rectangle size approaches that of a point, which is the intersection of BF and KE.

It is not so well known that, if <u>two cuboids</u> of square cross section  $(\varphi^{-1} \ge \varphi^{-1})$  are cut from the Golden Cuboid (broken lines, Fig. 2), the edge lengths of the remaining cuboid are in the same ratio as those of the original cuboid, viz.,  $1: \varphi^{-1}: \varphi^{-2} = \varphi: 1: \varphi^{-1}$ , so that this also is a Golden Cuboid,  $\varphi^{-3}$  times the size of the original.

The repetition of the decapitation process will lead to an indefinitely small Golden Cuboid located about a fixed point. The location of this point is left as an exercise to the reader.