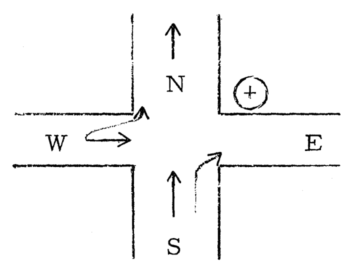
THE PROBLEM OF THE LITTLE OLD LADY TRYING TO CROSS THE BUSY STREET or FIBONACCI GAINED AND FIBONACCI RELOST

RICHARD BRIAN
San Jose State College, San Jose, California

It is no surprise to readers of this journal or to Fibonacci enthusiasts in general to find the numbers of the Fibonacci sequence popping up in the most peculiar places. This is an essay concerning an unusual situation in which these numbers appear in an interval of transience but are then overpowered by a linear function.



Consider the problem of an old lady standing on the northeast corner of the intersection of two one-way streets (one running north and the other running east) during rush hour. The traffic from the south may go east and north when its light is green but the traffic from the west may also go east and north when its light is green, hence a rather timid old lady might do well to bring a bag lunch if she anticipates such a situation.

Having viewed such a situation one evening I wondered if there might be some traffic pattern which would always allow the old lady to cross safely to any corner at any time.

Let us consider a network of one-way streets which alternate directions for both east and west and similarly north and south. If one is allowed to make a turn only at every other intersection, then one must always turn in the same direction. It is possible then to construct a traffic pattern in which one is allowed only to make turns to the right (see Figure 1).

A little study of this traffic pattern will show that one can drive to any location although it may require a trip around an extra block or two. But what of the old lady? Consider the corner letter AA. If she is standing on the northeast corner and wishes to cross to the west, she need only wait till the light stops the northbound traffic for the eastbound traffic cannot turn north. If she wishes to cross to the south,

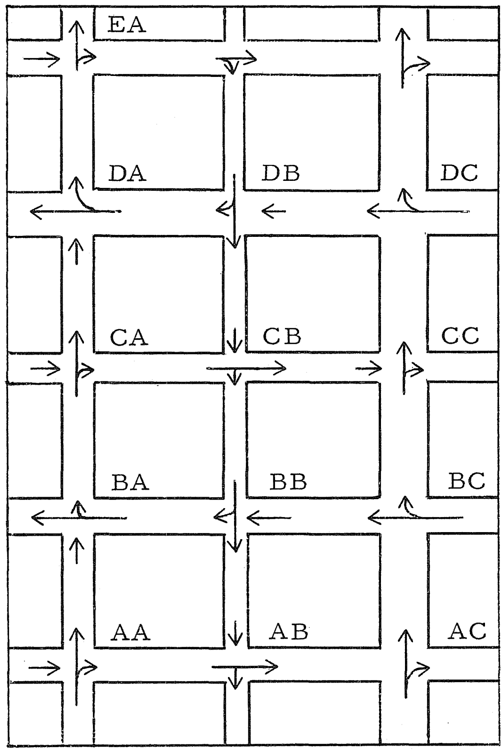


Figure 1

she cannot go directly but she can cross to the west, then to the south and finally to the east, still achieving her goal in complete safety.

Having solved the old lady's problem in every respect except convincing the traffic commission of the virtue of this scheme, I turned to other questions suggested by this same traffic scheme. Suppose one begins to drive north from corner (AA). How many blocks are accessible if one drives n (n = 1, 2, 3, 4, . . .) blocks? When one reaches corner (BA), going north, one must continue north since no turn is allowed northbound traffic here. When one reaches corner (CA) one may either turn to the east or proceed north and so on. Let us call f(n) the number of blocks which one adds to the total number of accessible blocks when driving on the nth block from corner (AA) then:

n	1	2	3	4	5	-	--	--	--
f(n)	1	1	2	3	5	-	--	--	-

Lo and behold f(n) appears to be the Fibonacci sequence. But there is a difficulty. One of the available paths after 5 blocks brings us back to corner (BA) travelling west. A turn to the north is allowed here but the block thus gained is one which we have already counted. Hence for $n = 6$ we have 7 new elements rather than 8 which is the next element of the Fibonacci sequence (F_n). This problem continues to plague us and if we count all the elements every time they occur, we do indeed get a Fibonacci sequence. However, if we do not count the duplications, our block acquisition sequence proceeds thus:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	--	--
f(n)	1	1	2	3	5	7	11	16	22	30	38	46	..	--	--
F_n	1	1	2	3	5	8	13	21	34	55	89	--	--	--	--

Now the question comes as to how fast this alteration takes place. Perhaps we notice that each of the last four entries differ by eight.

With this in mind consider the situation where one has a traffic pattern such that starting at corner (AA) one is allowed to go in any of the four directions and at the next corner any of the three remaining directions and so on. In this situation one acquires new elements at the rate $g(n) = 8n-4$.

It turns out that after the 9th step the acquisition of the new elements in the previous traffic pattern take on a linear form $f(n) = 8n-50$ $(n \geq 9)$.

In summary:

	L. O. L.	Traffic Pattern	Standard	Traffic Pattern
n	$f(n)$	$\sum_{i=1}^{n} f(i)$	$g(n)$	$\sum_{i=1}^{n} g(i)$
1	1	1	4	4
2	1	2	12	16
3	2	4	20	36
4	3	7	28	64
5	5	12	36	100
6	7	19	44	144
7	11	30	52	196
8	16	46		
9	22	68		
	$8n-50$ $(n \geq 9)$	$4n^2-46n+158$ $(n \geq 9)$	$8n-4$	$4n^2$

In this situation, then, the Fibonacci sequence appears only as a transient effect but such effects are, I think, relatively infrequent in purely abstract mathematical models.

XXXXXXXXXXXXXXX

(Continued from page 302.)

Thus every time that this sequence repeats there are only a possible 16 Fibonacci Numbers (the starred ones) out of 60 which both end in 1, 3, 7, or 9 and can be expressed as $6x\pm1$ and which just may be prime. Therefore we have established 16/60 or rather 4/15 of Euler's expression as an upper bound of the Fibonacci Prime Density.

XXXXXXXXXXXXXXX

NO WONDER NO SOLUTION

H-26 (Corrected) *Proposed by L. Carlitz, Duke University, Durham, N.C.*

Let $R_k = (b_{rs})$, where $b_{rs} = \binom{r-1}{k+1-s}$ $(r, s = 1, 2, \ldots, k+1)$ then show

$$R_k^n = \left(\sum_{j=1}^{s} \binom{r-1}{j-1} \binom{k+1-r}{s-j} F_{n-1}^{k+1-r-s+j} F_n^{r+s-2j} F_{n+1}^{j-1} \right).$$

XXXXXXXXXXXXXXX