# EXPANSIONS OF IN TERMS OF AN INFINITE CONTINUED FRACTION WITH PREDICTABLE TERMS

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(1) $$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots \text{ etc.}$$

Then, $$\pi = 4 - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \ldots \text{ etc.}$$

Whence, by identity of successive convergents, 4, 8/3, 52/15, 304/105, etc., in the above series, and in the following expansion, we have:

$$\pi = 4 - \cfrac{4}{3 + \cfrac{9}{2 + \cfrac{25}{2 + \cfrac{49}{2 + \cfrac{81}{2 + \text{etc.}}}}}}$$

i.e., $$\pi = \left\{ 4,\ -\frac{4}{3},\ \frac{9}{2},\ \frac{25}{2},\ \frac{49}{2},\ \text{etc.} \right\} .$$

(2) $$\frac{\pi}{4} - \frac{1}{2} = \sum_{1}^{\infty} (-1)^{n-1} \frac{1}{4n^2 - 1}, \quad \text{Jolley}$$

$$= \sum_{1}^{\infty} (-1)^{n} \frac{1}{(2n-1)(2n+1)}$$

$$\therefore \pi = 4\left\{ \frac{1}{2} + \frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \frac{1}{7 \cdot 9} + \text{etc.} \right\}$$

$$= \frac{10}{3} - \frac{4}{15} \quad \frac{4}{35} \quad \frac{4}{63} + \text{etc.}$$

whence, by identity of successive convergents, 10/3, 46/15, 334/105, 2946/945, etc., in the above series and in the following expansion, we have

$$\pi = 3 + \cfrac{1}{3 + \cfrac{12}{1 + \cfrac{16 - 1}{4 + \cfrac{36 - 1}{4 + \cfrac{64 - 1}{4 + \text{etc.}}}}}}$$

i.e., $$\pi = \left\{ 3,\ \frac{1}{3},\ \frac{12}{1},\ \frac{16-1}{4},\ \frac{36-1}{4},\ \ldots,\ \frac{(2k)^2-1}{4},\ \ldots \text{ etc.} \right\}$$

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