Again using the Binet formulas and the fact that $ab = -1$,

$$\begin{aligned} L_k L_{k+3m}^2 - L_{k+2m}^3 &= (a^k + b^k)(a^{k+3m} + b^{k+3m})^2 - (a^{k+2m} + b^{k+2m})^3 \\ &= (a^k + b^k)(a^{2k+6m} + 2(-1)^{k+m} + b^{2k+6m}) \\ &\quad - (a^{3k+6m} + 3(-1)^k a^{k+2m} + 3(-1)^k b^{k+2m} + b^{3k+6m}) \\ &= a^{3k+6m} + (-1)^k a^{k+6m} + 2(-1)^{k+m}(a^k + b^k) \\ &\quad + (-1)^k b^{k+6m} + b^{3k+6m} - a^{3k+6m} \\ &\quad - 3(-1)^k(a^{k+2m} + b^{k+2m}) - b^{3k+6m} \\ &= (-1)^k[(a^{k+6m} + b^{k+6m}) + 2(-1)^m(a^k + b^k) - 3(a^{k+2m} + b^{k+2m})]. \end{aligned}$$

Also

$$\begin{aligned} &5(-1)^k F_m^2 (L_{k+4m} + 2(-1)^m L_{k+2m}) \\ &= (-1)^k (a^m - b^m)^2 [(a^{k+4m} + b^{k+4m}) + 2(-1)^m(a^{k+2m} + b^{k+2m})] \\ &= (-1)^k (a^{2m} - 2(-1)^m + b^{2m})[(a^{k+4m} + b^{k+4m}) + 2(-1)^m(a^{k+2m} + b^{k+2m})] \\ &= (-1)^k [a^{k+6m} + b^{k+2m} + 2(-1)^m a^{k+4m} + 2(-1)^m b^k - 2(-1)^m a^{k+4m} - 2(-1)^m b^{k+4m} \\ &\quad - 4a^{k+2m} - 4b^{k+2m} + a^{k+2m} + b^{k+6m} + 2(-1)^m a^k + 2(-1)^m b^{k+4m}] \\ &= (-1)^k[(a^{k+6m} + b^{k+6m}) + 2(-1)^m(a^k + b^k) - 3(a^{k+2m} + b^{k+2m})]. \end{aligned}$$

This establishes the second formula.

Also solved by P. Bruckman, W. Janous, L. Kuipers, J. Spraggon, and the proposer.

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The Fibonacci Association and the University of Patras, Greece would like to announce their intentions to jointly sponsor an international conference on Fibonacci numbers and their applications. This conference is tentatively set for late August or early September of 1984. Anyone interested in presenting a paper or attending the conference should contact:

G. E. Bergum, Editor
The Fibonacci Quarterly
Department of Mathematics
South Dakota State University
Box 2220
Brookings, SD 57007-1297

Professor Andreas N. Philippou
Department of Mathematics
University of Patras
Patras, Greece