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THE GOLDEN MEAN IN THE SOLAR SYSTEM

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The mean distances of planets and satellites from their primary, divided by the next one out, bears a loose resemblance to the golden mean and the Fibonacci sequence. B. A. Read [6] explored this resemblance and related the deviation or offset of the planets from an exact Fibonacci spacing to the density of a planet and that of the next planet inward from it. However, when the aphelion and apogee distances are considered, the resemblance is no longer loose. Instead, as can be seen from the accompanying tables, the resemblance is close enough to reflect some underlying natural law. In Table 1, the observed aphelion distances of the planets from the sun are compared to distances calculated in direct proportion to the Fibonacci sequence as well as to distances calculated in proportion to the golden mean. The golden mean, 1.618034, an irrational number, is the limit that one Fibonacci number, divided by the preceding Fibonacci number, converges towards, which is equal to $(1 + \sqrt{5})/2$. Its reciprocal is 0.618034, which is the form of the golden mean used in this paper.

As can be seen from Table 1, Mercury deviates considerably from calculated distances, as would be expected from tidal interactions, as do the innermost satellites of Jupiter, Saturn, and Uranus in Table 2. The deviations of Jupiter and Saturn are not so easily dispensed with, and the gap between Jupiter and Saturn may be suggestive of a missing planet. However, Pluto's distance fits well, suggesting that Pluto is a normal member of the solar system rather than an asteroidal member. At the bottom of Table 1 is a statistical workup of the various calculated spacings compared with the observed spacings.

In the case of the planets, the Fibonacci sequence gives a better fit than the golden mean; however, the apogee distances of the satellites of Jupiter, Saturn, and Uranus fit the golden mean distances as well as the Fibonacci distances, as can be seen in Table 2. The Fibonacci and golden mean distances are calculated from assumed "true values" which are underlined. The asteroidal satellites of Jupiter form two families, the Himalia group consisting of Ananke, Carme, Sinape, and Pasiphae. The Himalia group satellites are close together and have a weighted apogee mean somewhat under the calculated value such that it appears more reasonable that they are fragments of a shattered moon rather than captured asteriodal objects. Likewise for the retrograde group; however, Ananke's inclusion may be doubtful and, if so, then the weighted mean would be 30360×10^3 Km, which fits better than the weighted mean for all four bodies.

Retrograde bodies may well be normal satellites or fragments of normal satellites. The break from direct to retrograde motion occurs at about the same value of the gravitational gradient for both Jupiter and Saturn. (The gravitational gradient is proportional to mass/distance cubed.) It would not surpirse these writers if both Uranus and Neptune were found to have outer retrograde satellites, and if planets beyond Pluto were found to be retrograde. In the case of the sun, Pluto lies farther out with respect to gravitational gradient than do the retrograde satellites of Jupiter and Saturn; thus, there is probably more to retrograde motion than gravitational gradient.

In Table 3, the aphelion and apogee distances are divided by the distance of the next body outward from the primary. For purposes of comparison and averaging over intermediate and skipped spacings, the resultant ratios in Table 1 are raised to appropriate exponents. Thus, it can be seen that the ratios of closely-spaced satellite orbits of Saturn correspond to the square root, 0.78615, of the golden mean reciprocal. In the statistical workup for the overall mean, the values for the innermost bodies, Mercury, Amalthea, the moonlets, and Miranda, were rejected since they would be the most subject to tidal forces. This workup yields a mean spacing ratio of 0.62103, which comes within 0.5% of the reciprocal of the golden mean. And if Phoebe and the "asteroidal" satellites of Jupiter are also rejected, the overall mean comes to 0.61877, which is within approximately 0.1% of that reciprocal. This golden mean orbital interval corresponds to a constant increase in the gravitational gradient by a factor

TABLE 1

COMPARISON OF OBSERVED APHELION DISTANCES WITH FIBONACCI AND GOLDEN MEAN RATIOS

Planet	Aphelion Distance from Sun $\times 10^6$ Km	Fibonacci Number	Distances Proportional to Fibonacci Numbers $\times 10^6$ Km: With Neptune at "True Value"	With Pluto at "True Value"	With an Adjusted Best Fit	Golden Mean Ratio Best Fit	Titius-Bode Law Numbers and Distances with Uranus at "True Value": Number	Distance $\times 10^6$ Km
MERCURY	69.86	1	50.98	51.22	51.88	61.04	4	61.3
VENUS	108.8	2	101.9	102.4	103.8	98.77	7	107.2
EARTH	152.1	3	152.9	153.6	155.6	159.8	10	153.2
MARS	249.1	5	254.9	256.1	259.3	258.6	16	245.1
ASTER.		8	407.8	409.7	414.8	418.4	28	248.9
JUPITER	815.8	13	662.7	665.8	674.1	677.0	52	796.4
		21	1070	1075	1089	1095	-	-
SATURN	1504	34	1733	1741	1763	1772	100	1532
URANUS	3002	55	2804	2817	2852	2868	196	3002
NEPTUNE	4537	89	4537	4558	4615	4640	-	-
PLUTO	7375	144	7340	7375	7467	7508	388	5943

The above $\frac{\text{Calculated Value}}{\text{Observed Value}} - 1$

	With Neptune at "True Value"	With Pluto at "True Value"	With an Adjusted Best Fit	Golden Mean Ratio Best Fit
MERCURY	-0.270	-0.267	-0.257	-0.126
VENUS	-0.063	-0.059	-0.046	-0.093
EARTH	0.006	0.010	0.023	0.051
MARS	0.023	0.028	0.041	0.038
JUPITER	-0.188	-0.184	-0.174	-0.170
SATURN	0.152	0.157	0.172	0.178
URANUS	-0.066	-0.062	-0.050	-0.045
NEPTUNE	0.000	0.005	0.017	0.023
PLUTO	0.005	0.000	0.013	0.018
Sum, less Mercury	-0.131	-0.109	-0.002	0.000

The aphelion distances were taken from Joseph Armento's compilation [1]. The Titius-Bode law relationship, which works best with mean distances is shown for comparison only.

TABLE 2

COMPARISON OF THE APOGEE DISTANCES OF THE SATELLITES OF JUPITER, SATURN, AND URANUS WITH THE FIBONACCI AND GOLDEN MEAN RATIOS

(The satellites are listed in order of increasing apogee distances.)

Satellite	Mean Distance $\times 10^3$ Km	Eccen-tricity	Inclina-tion	Apogee Distance $\times 10^3$ Km	Fibonacci Number	Distance Proportional to Fibonacci	Distance Proportional Golden Mean
JUPITER							
Amalthea	181.3	0.003	0.4	181.8	1	233.7	256.3
Io	421.6	0.000	0.0	241.6	2	447.3	414.7
Europa	670.9	0.0001	0.5	671.0	3	<u>671.0</u>	<u>671.0</u>
Ganymede	1070	0.0014	0.2	1072	5	1118	1086
Callisto	1883	0.0074	0.2	1897	8	1789	1757
Leda	11110	0.1478	26.7	12750			
Lysithea	11710	0.1074	29	12970			
Himalia	11470	0.1580	28	13280			
Elara	11720	0.2072	28	14180			
Weighted mean of the above four . . .				13370	55	12300	12040
Ananke	20700	0.169	147	24200			
Carme	22350	0.207	163	26980			
Sinape	23700	0.275	157	30220			
Pasiphae	23300	0.410	148	32850			
Weighted mean of the above four . . .				29750	144	32210	31530
SATURN							
					1		
Weighted mean apogee distance of A and B ring moonlets 1980S28, S27, S26, S3, and S1				150	$\sqrt{2}$	138	145
Mimas	186	0.020	1.5	190	2	196	184
Enceladus	238	0.005	0.0	239	$2\sqrt{3/2}$	240	235
Tethys	295	0.000	1.1	295	3	294	298
Dione	378	0.002	0.0	379	$3\sqrt{5/3}$	<u>379</u>	<u>379</u>
Rhea	528	0.001	0.3	529	5	489	482
Titan	1223	0.029	0.3	1258	13	1272	1267
Hyperion	1484	0.104	0.6	1638	$13\sqrt{21/13}$	1616	1605
Iapetus	3562	0.028	14.7	3662	34	3327	3304
Phoebe	12960	0.163	150	15070	144	14090	14000
URANUS							
Miranda	130.5	0.00	0.0	130.5	$1/\sqrt{2}$	137.0	145.4
Ariel	191.8	0.003	0.0	192.4	2	193.8	185.0
Umbriel	267.2	0.004	0.0	268.3	3	290.6	299.4
Titania	483.4	0.002	0.0	484.4	5	<u>484.4</u>	<u>484.4</u>
Oberon	586.3	0.001	0.0	586.9	$5\sqrt{8/5}$	612.8	616.2

Weighted means were found by multiplying apogee distances by radii cubed. In the case of the Himalia group, the diameters of Leda, Lysithea, Himalia, and Elara are 8, 19, 170, and 80 Km, respectively. In the retrograde group, the diameters of Ananke, Carme, Sinape, and Pasiphae are 17, 24, 21, and 27 Km, respectively.

Assumed "true values" from which calculations were started are underlined.

Satellite data were obtained from Patrick Moore's compilation [2]. A and B moonlets distance calculated from Robert Burnham's compilation [3].

TABLE 3

ORBITAL RATIOS

Planet or Satellite	$\frac{d_1}{d_2}$	Exponent y	$\left(\frac{d_1}{d_2}\right)^y$
Mercury	0.64180	1	0.64180
Venus	0.71579	1	0.71579
Earth	0.61043	1	0.61043
Mars	0.30537	1/2	0.53260
Jupiter	0.54224	1/2	0.73637
Saturn	0.50112	1	0.50112
Uranus	0.66177	1	0.66177
Neptune	0.61516	1	0.61516
Amalthea	0.43121	1	0.43121
Io	0.62832	1	0.62832
Europa	0.62622	1	0.62622
Ganymede	0.56484	1	0.56484
Callisto	0.14191	1/4	0.61374
Himalia Group	0.44038	1/2	0.66361
Moonlets	0.78947	2	0.62327
Mimas	0.79498	2	0.63199
Enceladus	0.81017	2	0.65637
Tethys	0.77836	2	0.60585
Dione	0.71645	2	0.51329
Rhea	0.42051	1/2	0.64847
Titan	0.76801	2	0.58984
Hyperion	0.44730	2/3	0.58488
Iapetus	0.24300	1/3	0.62400
Miranda	0.67827	2	0.46006
Ariel	0.71711	1	0.71711
Umbriel	0.55388	1	0.55388
Titania	0.82535	2	0.68121
Mean ratio with Mercury, Amalthea, moonlets, and Miranda excluded			0.62103

This table shows the aphelion and apogee distances, d_1, of planets and satellites divided by the distance, d_2, of the next body outward from the primary. The ratios are raised to various powers for purposes of averaging over intermediate spacings or skipped spacings.

of 4.236068, the cube of the golden mean, which is equal to $2 + \sqrt{5}$, going from one orbit inward to the next orbit nearer to the primacy.

Concerning the motions of Mercury and Venus, Robert R. Newton [4] has come up with some interesting observations. He has carefully analyzed astronomical observations since Babylonian times and has noted that Mercury has been persistently gaining energy and, likewise, Venus to a lesser extent. The angular accelerations he has come up with, in seconds of arc per century squared are: Mercury, 4.1520; Venus, 1.6225. These numbers are maximum values; thus, the true values are probably one-half or less of these numbers. These numbers are of the right magnitude to account for the deviation from golden mean positions for these planets. Robert R. Newton [4] has noted a small increase in Saturn's angular motion, but not enough to account for the observed discrepancy. No change has been noted for Jupiter. Possibly the explanation lies in the large mass of Jupiter and Saturn.

The authors conclude that there is some underlying law involving gravitation and the golden mean that determines both aphelion and apogee distances. With respect to some underlying gravitational principle, R. Louise [5] remarked: "that satellite systems mimic the planetary system suggests some possible unsuspected property of gravitation."

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