A FIBONACCI-LIKE SEQUENCE OF ABUNDANT NUMBERS

CHARLES R. WALL
Trident Technical College, Charleston, SC 29411
(Submitted April 1983)

Let $\sigma(n)$ denote the sum of the divisors of n. An integer n is said to be *abundant* if $\sigma(n) > 2n$, *perfect* if $\sigma(n) = 2n$, or *deficient* if $\sigma(n) < 2n$. It is known [2] that if the greatest common divisor of the integers a and b is deficient, then there exist infinitely many deficient integers $n \equiv a \pmod{b}$. Fibonacci buffs might expect an analogous result for generalized Fibonacci numbers, something along the lines of "if $x_{n+1} = x_n + x_{n-1}$ and gcd (x_1, x_2) is deficient, then the sequence $\{x_n\}$ contains infinitely many deficient terms." In this note we shatter any such expectations by constructing a Fibonacci-like sequence $\{x_n\}$ with all terms abundant and having gcd (x_1, x_2) deficient.

Vital to the construction are two easily proved theorems:

(1) Any multiple of an abundant number is abundant.

(2) If p is an odd prime, then $2^a p$ is abundant if $p < 2^{a+1} - 1$,
perfect if $p = 2^{a+1} - 1$,
and deficient if $p > 2^{a+1} - 1$.

Graham [1] defined a Fibonacci-like sequence by

$$g_1 = 1786\ 772701\ 928802\ 632268\ 715130\ 455793,$$

$$g_2 = 1059\ 683225\ 053915\ 111058\ 165141\ 686995,$$

and $g_{n+1} = g_n + g_{n-1}$. Graham's sequence has the remarkable property that even though gcd $(g_1, g_2) = 1$, every term is composite. More specifically, every term is divisible by at least one of the primes

(3) 2, 3, 5, 7, 11, 17, 19, 31, 41, 47, 53, 61, 109, 1087, 2207, 2521, 4481, 5779.

Now, define a sequence $\{x_n\}$ by

$$x_n = 2^{12} 8209 \cdot g_n,$$

where $\{g_n\}$ is Graham's sequence. Since $5779 < 2^{13} = 8192$, $2^{12}q$ is abundant for each odd q listed in (3), and $2^{13}8209$ is abundant since $8209 < 2^{14} - 1$. Therefor, each x_n is abundant. But

$$\gcd(x_1, x_2) = 2^{12}8209$$

is deficient since $8209 > 2^{13} - 1$.

Clearly, in the construction above, we may replace 8209 by any prime p such that $2^{13} < p < 2^{14}$.

REFERENCES

1. R. L. Graham. "A Fibonacci-Like Sequence of Composite Numbers." *Math. Mag.* 37 (1964):322-34.
2. C. R. Wall. Problem proposal E3002. *Amer. Math. Monthly* 90 (1983):400.

◆◇◆◇◆