



# The Fibonacci Quarterly

THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION

## TABLE OF CONTENTS

On Primitive Pythagorean Triangles with Equal Perimeters . . . <i>Leon Bernstein</i>	2
On a Generalization of the Fibonacci Sequence in the Case of Three Sequences. . . . . <i>Krassimir T. Atanassov</i>	7
On the Periods of the Fibonacci Sequence Modulo $m$ . . . . . <i>Amos Ehrlich</i>	11
How Many 1's Are Needed? . . . . . <i>Daniel A. Rawsthorne</i>	14
Optimal Spacing of Points on a Circle . . . . . <i>Tony van Ravenstein</i>	18
Congruence Relations for $k^{\text{th}}$ -Order Linear Recurrences . . . <i>Lawrence Somer</i>	25
Referees . . . . .	31-32
A Recurrence Restricted by a Diagonal Condition: Generalized Catalan Arrays. . . . . <i>Robert A. Sulanke</i>	33
Fibonacci-Like Matrices . . . . . <i>John F. Morrison</i>	47
Representations for Real Numbers Via $k^{\text{th}}$ Powers of Integers. . . . . <i>Arnold Knopfmacher and John Knopfmacher</i>	49
Congruences for Numbers of Ramanujan. . . <i>F. T. Howard and E. K. Hayashi</i>	61
On a Certain Sequence of Quotients of a Sequence. . . . . <i>Alejandro Necochea and Larry Bennett</i>	70
The Alpha and the Omega of the Wythoff Pairs . . . . . <i>J. C. Turner</i>	76
Elementary Problems and Solutions. . . . . <i>Edited by A. P. Hillman</i>	87
Advanced Problems and Solutions. . . . . <i>Edited by Raymond E. Whitney</i>	92

VOLUME 27

FEBRUARY 1989

NUMBER 1

## PURPOSE

The primary function of **THE FIBONACCI QUARTERLY** is to serve as a focal point for widespread interest in the Fibonacci and related numbers, especially with respect to new results, research proposals, challenging problems, and innovative proofs of old ideas.

## EDITORIAL POLICY

**THE FIBONACCI QUARTERLY** seeks articles that are intelligible yet stimulating to its readers, most of whom are university teachers and students. These articles should be lively and well motivated, with new ideas that develop enthusiasm for number sequences or the exploration of number facts. Illustrations and tables should be wisely used to clarify the ideas of the manuscript. Unanswered questions are encouraged, and a complete list of references is absolutely necessary.

## SUBMITTING AN ARTICLE

Articles should be submitted in the format of the current issues of the **THE FIBONACCI QUARTERLY**. They should be typewritten or reproduced typewritten copies, that are clearly readable, double spaced with wide margins and on only one side of the paper. The full name and address of the author must appear at the beginning of the paper directly under the title. Illustrations should be carefully drawn in India ink on separate sheets of bond paper or vellum, approximately twice the size they are to appear in print.

Two copies of the manuscript should be submitted to: **GERALD E. BERGUM, EDITOR, THE FIBONACCI QUARTERLY, DEPARTMENT OF COMPUTER SCIENCE, SOUTH DAKOTA STATE UNIVERSITY, BOX 2201, BROOKINGS, SD 57007-0194.**

Authors are encouraged to keep a copy of their manuscripts for their own files as protection against loss. The editor will give immediate acknowledgment of all manuscripts received.

## SUBSCRIPTIONS, ADDRESS CHANGE, AND REPRINT INFORMATION

Address all subscription correspondence, including notification of address change, to: **RICHARD VINE, SUBSCRIPTION MANAGER, THE FIBONACCI ASSOCIATION, SANTA CLARA UNIVERSITY, SANTA CLARA, CA 95053.**

Requests for reprint permission should be directed to the editor. However, general permission is granted to members of The Fibonacci Association for noncommercial reproduction of a limited quantity of individual articles (in whole or in part) provided complete reference is made to the source.

Annual domestic Fibonacci Association membership dues, which include a subscription to **THE FIBONACCI QUARTERLY**, are \$30 for Regular Membership, \$40 for Sustain Membership, and \$67 for Institutional Membership; foreign rates, which are based on international mailing rates, are somewhat higher than domestic rates; please write for details. **THE FIBONACCI QUARTERLY** is published each February, May, August and November.

All back issues of **THE FIBONACCI QUARTERLY** are available in microfilm or hard copy format from **UNIVERSITY MICROFILMS INTERNATIONAL, 300 NORTH ZEEB ROAD, DEPT. P.R., ANN ARBOR, MI 48106.** Reprints can also be purchased from **UMI CLEARING HOUSE** at the same address.

1989 by

© The Fibonacci Association

All rights reserved, including rights to this journal issue as a whole and, except where otherwise noted, rights to each individual contribution.

# *The Fibonacci Quarterly*

Founded in 1963 by Verner E. Hoggatt, Jr. (1921-1980)  
and Br. Alfred Brousseau (1907-1988)

*THE OFFICIAL JOURNAL OF THE FIBONACCI ASSOCIATION  
DEVOTED TO THE STUDY  
OF INTEGERS WITH SPECIAL PROPERTIES*

## **EDITOR**

GERALD E. BERGUM, South Dakota State University, Brookings, SD 57007-0194

## **ASSISTANT EDITORS**

MAXEY BROOKE, Sweeny, TX 77480  
JOHN BURKE, Gonzaga University, Spokane, WA 99258  
PAUL F. BYRD, San Jose State University, San Jose, CA 95192  
LEONARD CARLITZ, Duke University, Durham, NC 27706  
HENRY W. GOULD, West Virginia University, Morgantown, WV 26506  
A.P. HILLMAN, University of New Mexico, Albuquerque, NM 87131  
A.F. HORADAM, University of New England, Armidale, N.S.W. 2351, Australia  
FRED T. HOWARD, Wake Forest University, Winston-Salem, NC 27109  
DAVID A. KLARNER, University of Nebraska, Lincoln, NE 68588  
RICHARD MOLLIN, University of Calgary, Calgary T2N 1N4, Alberta, Canada  
JOHN RABUNG, Randolph-Macon College, Ashland, VA 23005  
DONALD W. ROBINSON, Brigham Young University, Provo, UT 84602  
LAWRENCE SOMER, Catholic University of America, Washington, D.C. 20064  
M.N.S. SWAMY, Concordia University, Montreal H3C 1M8, Quebec, Canada  
D.E. THORO, San Jose State University, San Jose, CA 95192  
WILLIAM WEBB, Washington State University, Pullman, WA 99163

## **BOARD OF DIRECTORS OF THE FIBONACCI ASSOCIATION**

CALVIN LONG (President)  
Washington State University, Pullman, WA 99163  
G.L. ALEXANDERSON  
Santa Clara University, Santa Clara, CA 95053  
PETER HAGIS, JR.  
Temple University, Philadelphia, PA 19122  
MARJORIE JOHNSON (Secretary-Treasurer)  
Santa Clara Unified School District, Santa Clara, CA 95051  
JEFF LAGARIAS  
Bell Laboratories, Murray Hill, NJ 07974  
LESTER LANGE  
San Jose State University, San Jose, CA 95192  
THERESA VAUGHAN  
University of North Carolina, Greensboro, NC 27412